Cellular Automata (CA)

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Presented by

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Questions

1. What is quiescent state of the cellular automata?

2. How many possible transition rules are there, if there are state set of 2 elements and a neighborhood composed of 2 cells?

3. Why random initial state is used in Langton’s approach?

Xiande Zhang

- First year master's student in Computer Engineering
  - Graduate Research Assistant in the MoSIS Lab
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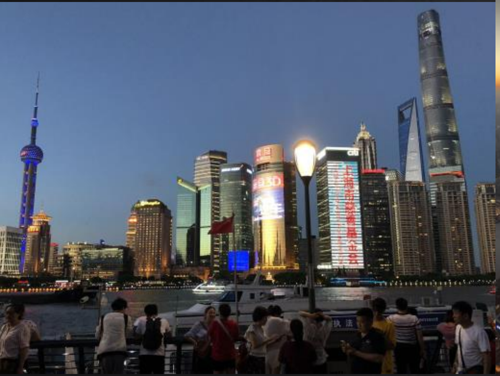
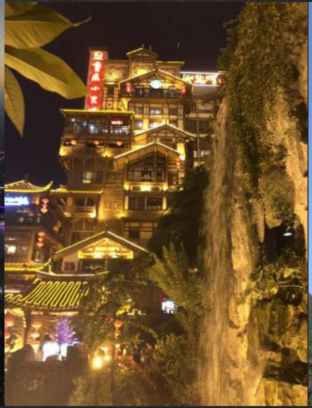
- B.S., Computer Engineering, University of Tennessee, Knoxville (2017-2021)
- Hometown: Fu Zhou, China

Food

Traveling





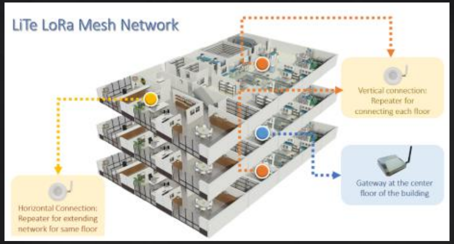
Patrick, HON-MAN LAU

- Master student, Electrical Engineering
  - Graduate Research Assistant @ ICL & MoSIS Lab
  - Advisors: Dr. Stanimire Tomov, Dr. Jian Liu
  - Research interests: Edge computing, machine learning

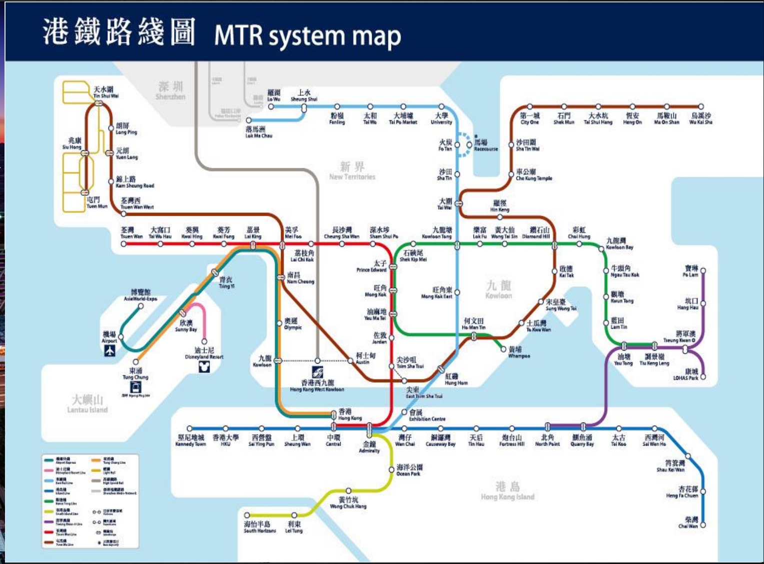
- M.S., Electrical and Electronic Engineering, University of Hong Kong (HKU)
- B.S., Electronic and Communication Engineering, City University of Hong Kong (CityU)

- Co-founder of a startup company in Hong Kong
  - Smart building solution

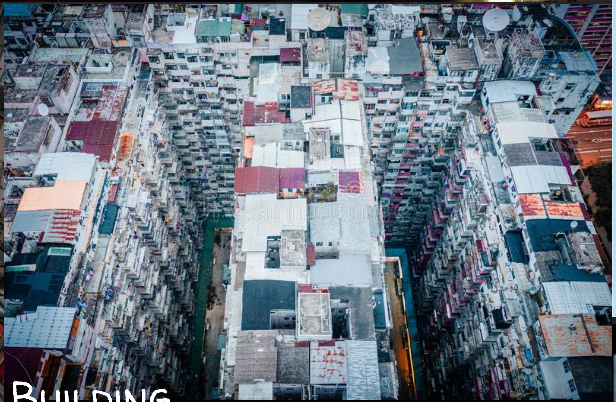
- Hometown: Hong Kong



Victoria Harbor

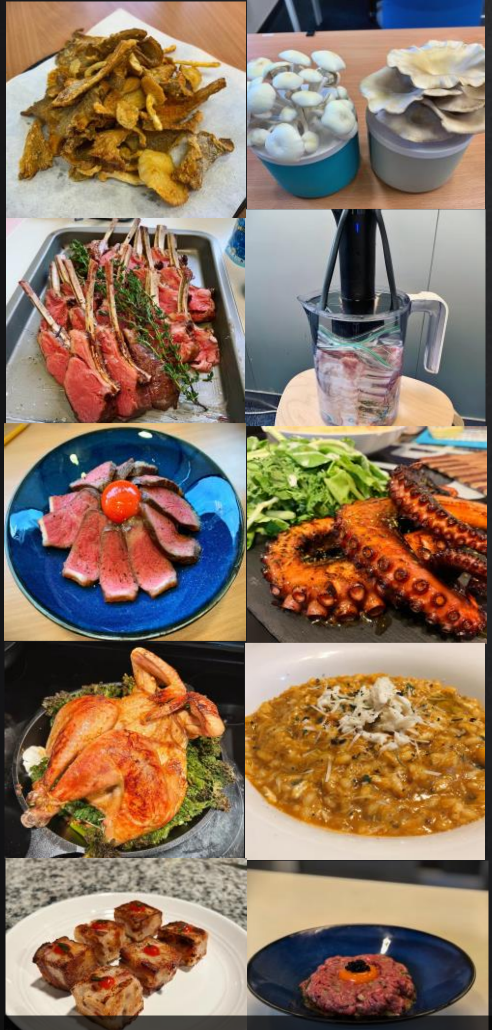


Monster Building

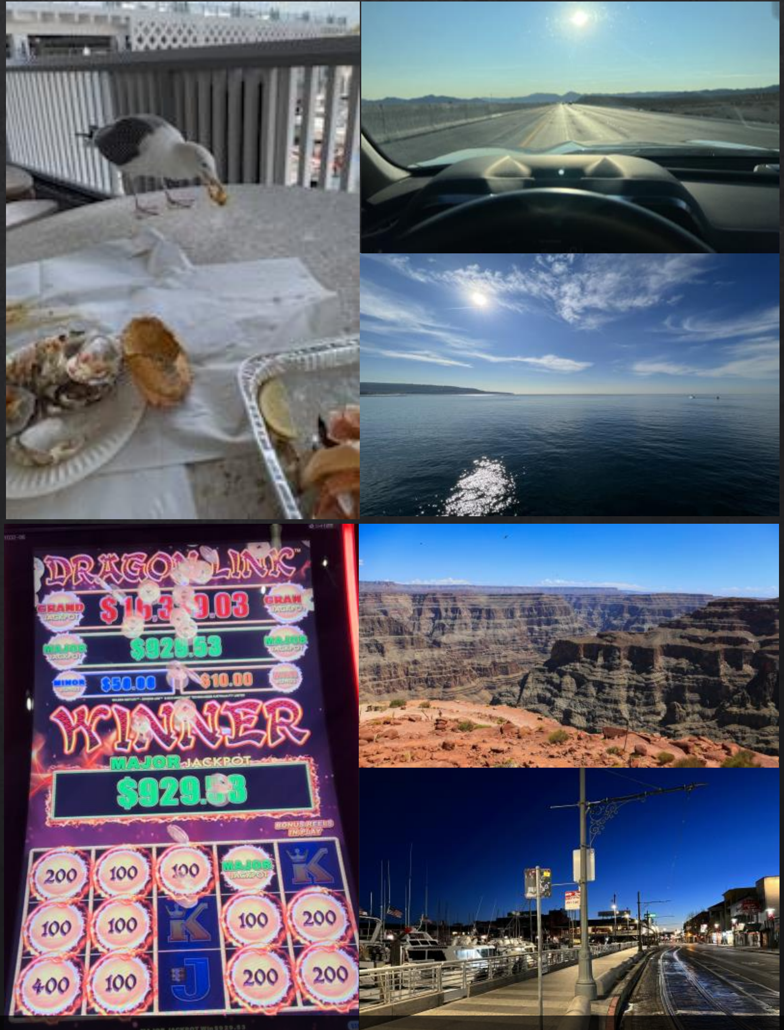


Transformers: Age of Extinction, Michael Bay, 2014

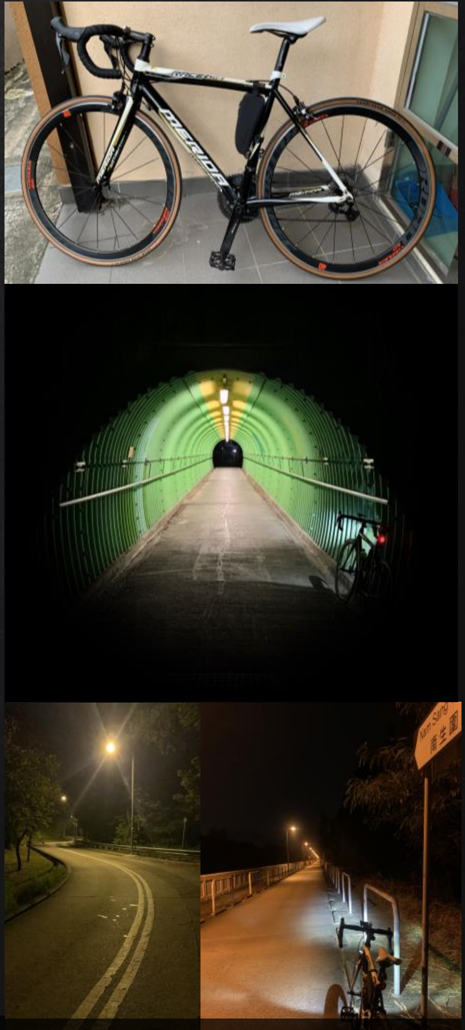
Godzilla vs. Destoroyah (1995)



Cooking



Traveling



Biking

Outline

Overview
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Algorithms – CA
Algorithm – Game of Life
Algorithm – Edge of Chaos
Applications
Open Issues
References
Discussion

Overview

Overview #1

Cellular System

- any system composed of interconnected, interacting units or entities called cells
- these cells can exchange information, resources, or influence with one another
- the behavior of the system emerges from the collective interactions of the cells

Cellular Automata (CA)

- Specific type of cellular system
- Uses discrete mathematical models to study and simulate complex systems
- Consist of a grid of cells, each of which can take on a finite set of states, and they evolve over time following a set of deterministic rules

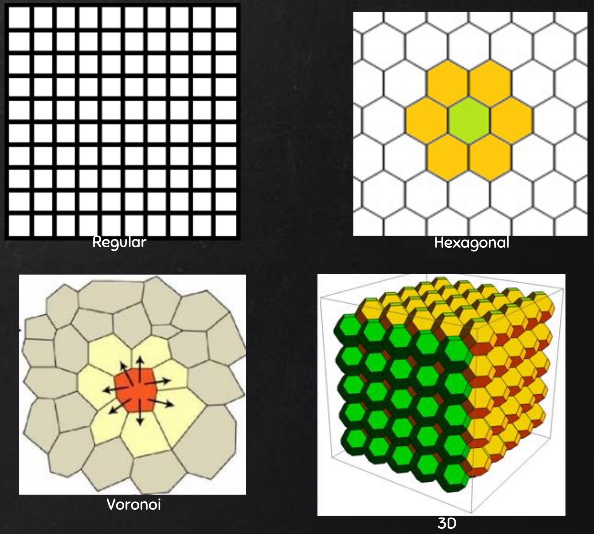
State

- Each cell have a state representing its current condition
- May change over time based over the conditions of the entire system
- The state of cell is how it retains memory
- **Quiescent state**: Inactive or resting condition

Overview #2

Cellular Space & Time

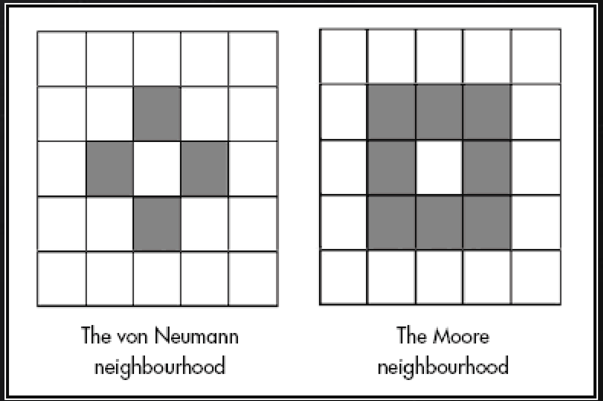
- 1D, 2D, or even higher-dimensional (rarely more than 3D)
- Finite / infinite size
- Cellular space: Arrangement of cells in the grid
- Discrete time step



Ref: https://unigis-salzburg.github.io/Opt_Spatial-Simulation/cellular-automata.html

Neighborhood

- includes the surrounding cells that influence its state (0 or 1)
- Different types of neighborhoods can be used
  - von Neumann
  - Moore neighborhoods
- the choice of neighborhood impact the behavior of the CA



Ref: https://artificialnature.net/courses/datt4950/cellular.html

Overview #3

State Transition Function

- The state transition function determines how a cell's state will change based on the states of its neighboring cells, and current state
- This function can be simple or complex, and it is applied to all cells simultaneously at each time step
- The position of cell or time value can also be related

Transition Table

- A representation of the state transition function in the form of a table
- Helps to define the behavior of a cellular automaton or any other state-based system
- The table lists all possible neighborhood configurations and the corresponding new states for the central cell

History

Early Foundations (1943 - 1956)

Warren McCulloch, Walter Pitts

- While the term "cellular automata" was not yet coined, this paper laid the groundwork for the field by introducing a simple model of artificial neurons and a mathematical formalism for their behavior

Paper:

- A Logical Calculus of the Ideas Immanent in Nervous Activity (1943)

Alan Turing

- Discusses reaction-diffusion systems, a type of continuous-space cellular automaton, as a model for pattern formation in biological systems

Paper:

- The Chemical Basis of Morphogenesis (1952)

John von Neumann

- Formally introducing the concept of CA and self-replication

Paper:

- The Behavior of Automata with a Large Number of States (1956)

Initial Development and Exploration (1960 – 1970)

Edward Fredkin

- Proposes the idea of digital mechanics and investigates CA as a model for the fundamental laws of physics

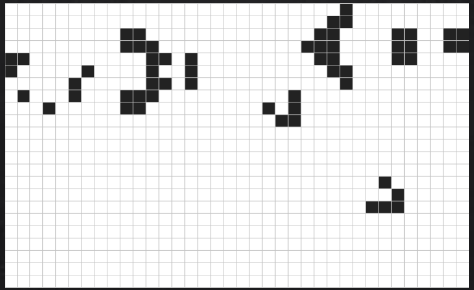
Paper:

- An Information-Theoretic Approach to Digital Mechanics (1969)

John Conway

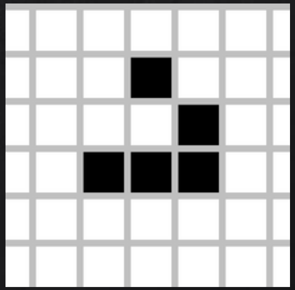
- "Game of Life"
- 2D cellular automaton
- Becomes widely popular and further spurs interests in CA

Gosper Glider Gun



Ref: https://blog.datawrapper.de/game-of-life/

Glider



Ref: https://sweetpeagallery.ca/blogs/editorial/the-game-of-life-early-generative-art

Classification & Characterization (1980 – 1990)

Stephen Wolfram

- Developed a classification system
- Systematically exploring CA's properties
- Classifying them into four distinct classes based on their behavior

Paper:

– Statistical mechanics of cellular automata (1983)
– Universality and complexity in cellular automata (1984)
– Cellular automata as models of complexity (1984)

Class	Behavior
Class 1	Uniformity
Class 2	Stability, Periodicity
Class 3	Randomness, Chaos
Class 4	Complexity, Computation

Chris Langton

- Introduced the concept of "edge of chaos"
- Highlighting the significance of system operating in the transition region between order and chaos

Applications & Expanding Horizons (2000s – Present)

Stephen Wolfram

- Published a comprehensive book, “A New Kind of Science” in 2002
- Including the classification of CA into four distinct classes based on their behavior

Gérard Vichniac

- Introduces the concept of “quasi-species” in CA
- Exploring the concept of “evolution” with CA systems

Melanie Mitchell, James P. Crutchfield

- Investigates the concept of the “edge of chaos” in the context of cellular automata
- Provided a deeper understanding of the properties and behaviors of these systems in the critical region between order and chaos

Paper:

– Revisiting the Edge of Chaos: Evolving Cellular Automata to Perform Computations (1993)

Algorithm

Cellular Automata

Cellular Automata

- Time variable: Discrete
- Neighborhood: Finite
- State set (S): Finite $S = \{0, ..., k-1\}$
- Quiescent state: $S_0 = 0$
- All cells are updated synchronously

The transition rule is a function that operates deterministically

- $S_i(t+1) = \varphi(S_j(t) : j \in N_i)$
  - Determines how the state of each cell changes over time
  - Homogeneous function:
    - Cell's neighborhood
    - Cell's position
- Simple rule example:
    - Active neighbor
    - Inactive state
    - Activation condition

Cellular Automata

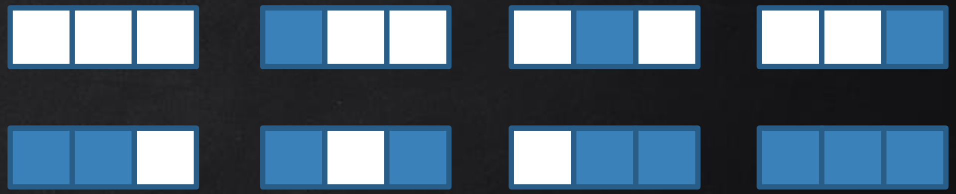
- Transition table:
  - Based on the transition rule function
    - It represents the behavior of a state machine, detailing the transitions between states based on input and conditions
    - So, when a cellular automation has a state set of k elements and a neighborhood composed of n cells, the total number of possible configurations is k^n

Example:

if $k = 2$ and $n = 3$,

Possible configurations = 2^3 = 8

Possible transition rules = 2^{2^3} = 256



Initial state (seed)

- Transition rule

- Time-based simulation

Simulation stopping conditions

- Fixed time
- Cyclical behavior

Cellular Automata

Special Cellular automata rules:

Totalistic: $S_i(t+1) = \varphi(\sum_{j \in N_j} S_j(t))$

- New cell state:
  - Function φ based on neighbors' states
- Neighbor sum:
  - 8 adjacent cells in 2D grid

Outer Totalistic: $S_i(t+1) = \varphi(\mathrm{Si}(\mathrm{t}), \sum_{j \in N_j} S_j(t))$ when $j \neq i$

- New cell state:
  - Function φ of current state and neighbors' states
- Neighbor sum:
  - Adjacent cells, excluding cell i

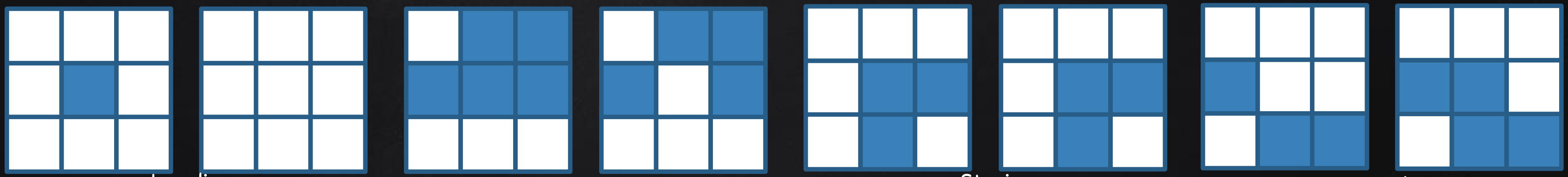
- Totalistic rules:
  - Symmetric (order-independent)
  - Null state quiescent rule (stability)

Algorithm

Game of Life

Game of Life

- Zero-player game: Evolution determined by initial state, no human input required.
- Interaction: Create initial configuration and observe its evolution.
- RULES:
  - Game of Life universe:
    - Infinite two-dimensional grid
    - Cells are either alive or dead
    - Eight neighbors (horizontal, vertical, diagonal)
  - Cell interaction rules:
    - Loneliness: live cells with <2 neighbors die
    - Overcrowding: live cells with >3 neighbors die
    - Stasis: live cells with 2–3 neighbors remain alive
    - Reproduction: Any dead cell with exactly 3 neighbors comes to life

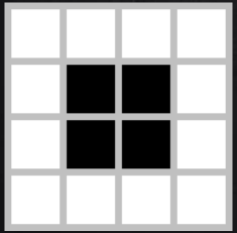


Game of Life

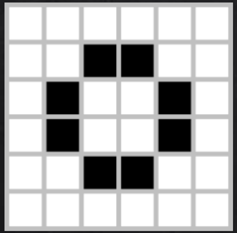
In Cellular Automata, like Conway's Game of Life, configurations show distinct behaviors, such as still-life and oscillators.

- Still-life configurations:
  - Remain static over time
  - Examples: Block, Pond, Bee-hive

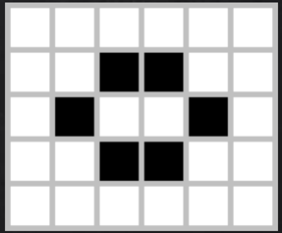
- Oscillators:
  - Cycle through finite states, return to original configuration
  - Examples: Blinker, Clock II.



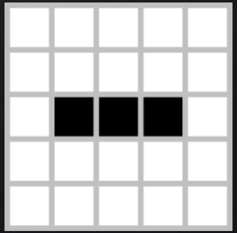
Block



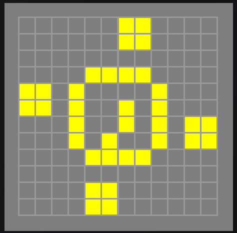
Pond



Bee-hive



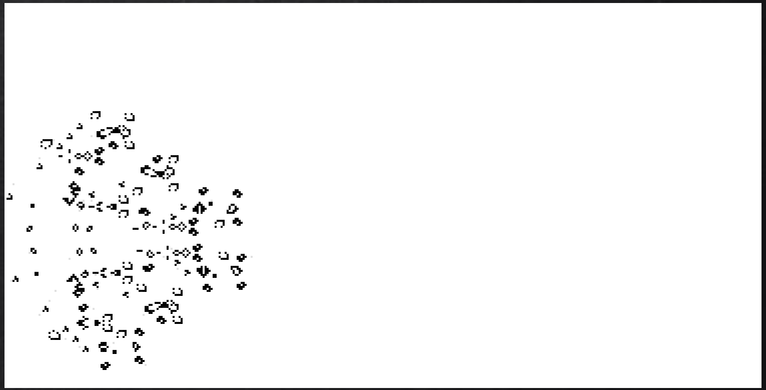
Blinker

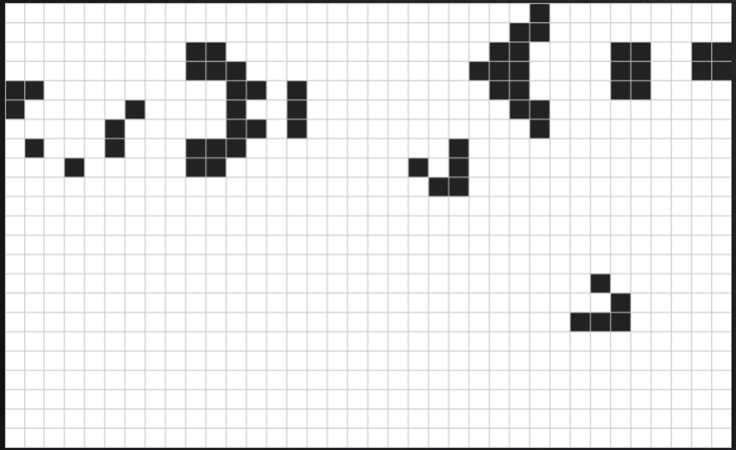


Clock II

Game of life

- Moving objects:
  - Glider (diagonal motion)
  - Information transmission across grid
- Information processing objects:
  - Eater (consumes or stabilizes objects)
  - Simulates complex systems and information processing





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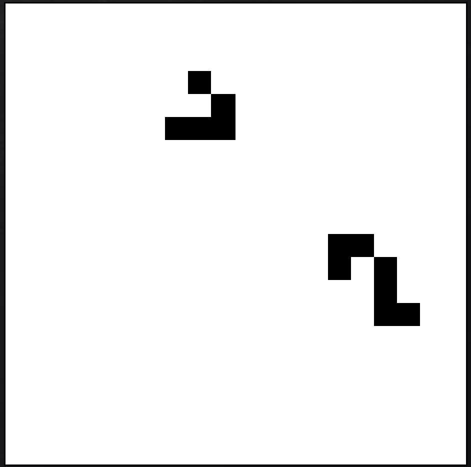


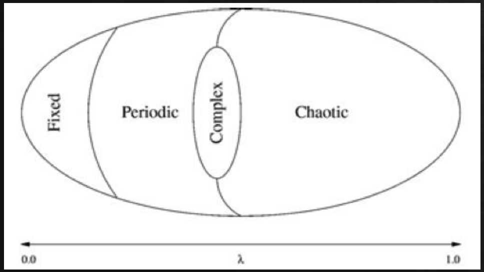
https://users.math.yale.edu/public_html/People/frame/Fractals/CA/CAPatterns/Life/LifeAnd/EaterMovie.html

Algorithm

Edge of Chaos

Edge of Chaos

- There are total of four Wolfram's cellular automata classes
  - Class I: Progresses to a stable, uniform state
    - End state is a fixed point
  - Class II: Progresses to isolated, repeating structures
    - End state is a cyclical pattern
  - Class III: Produces unpredictable, non-repeating patterns
    - Characterized by a strange attractor, indicating chaotic behavior
  - Class IV: Generates intricate patterns of localized structure
    - Characterized by long transients, with no equivalent in dynamic systems



http://node99.org/tutorials/eoc/

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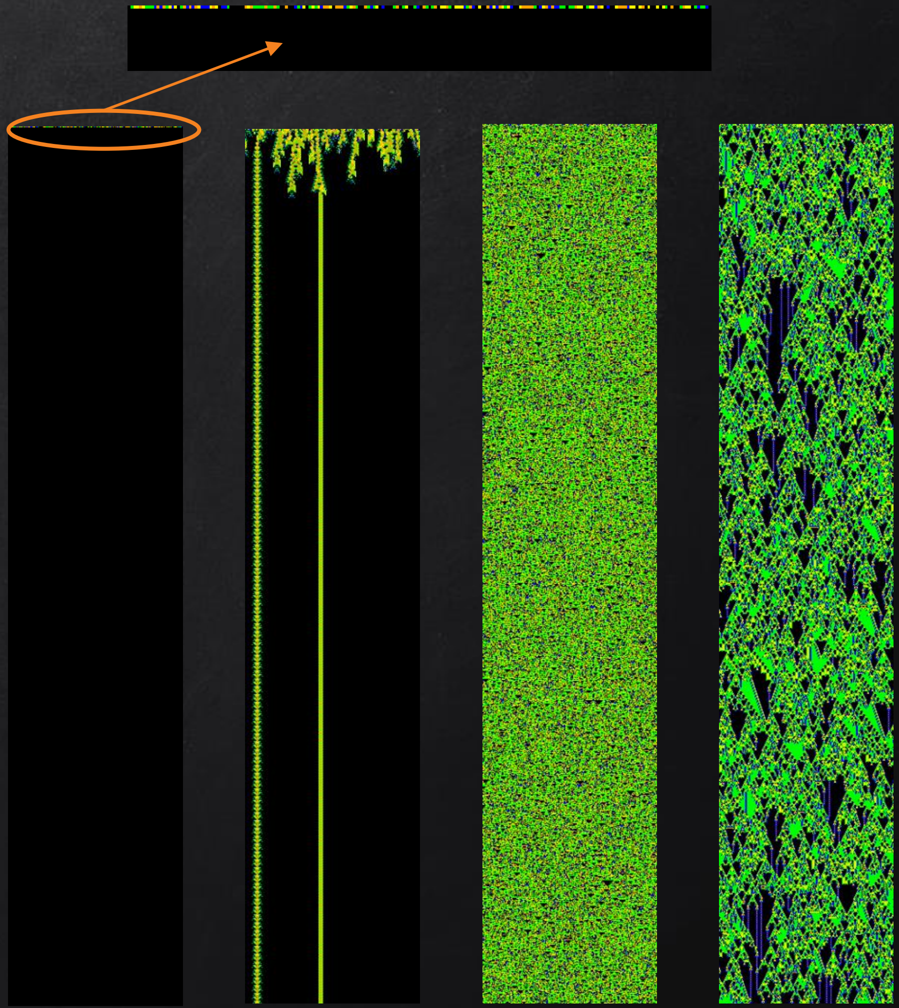
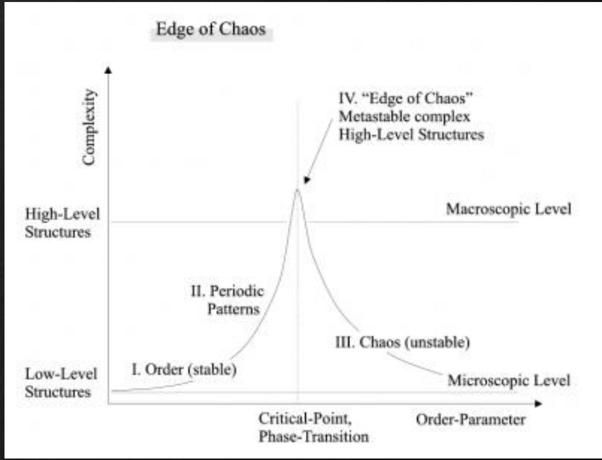


Edge of Chaos

Langton's Approach:

- Explore 1D CA by:
  - Applying random transition rules
  - Initiating the system in random states
    - Some specific starting conditions could result in unusual or atypical behavior.
    - Using a **random initial state** essentially allows the Cellular Automata (CA) to operate simultaneously on a variety of initial states.
    - This process explores the emergence of structured order from seemingly random conditions.
- Gradually modify a straightforward parameter that characterizes the rule
- Analyze the overall behavior based on Wolfram's classification scheme



Ref: https://wiki.cas-group.net/index.php?title=File:Edge_of_Chaos.png

Edge of Chaos

Langton's Lambda: $\lambda = (T-n_q)/T$

- If all configurations lead to a passive or inactive state: $\lambda = 0$
- If no configurations lead to a passive or inactive state: $\lambda = 1$
- If every state appears with equal frequency: $\lambda = 1 - 1 / K$
- A kind of gauge of "responsiveness" or "sensitivity" to change.

Radius	r
Number of states	K
Size of the neighborhood	N = 2r + 1
Number of entries in the table	T= kN
Number mapping to quiescent state	n_q

Entropy:

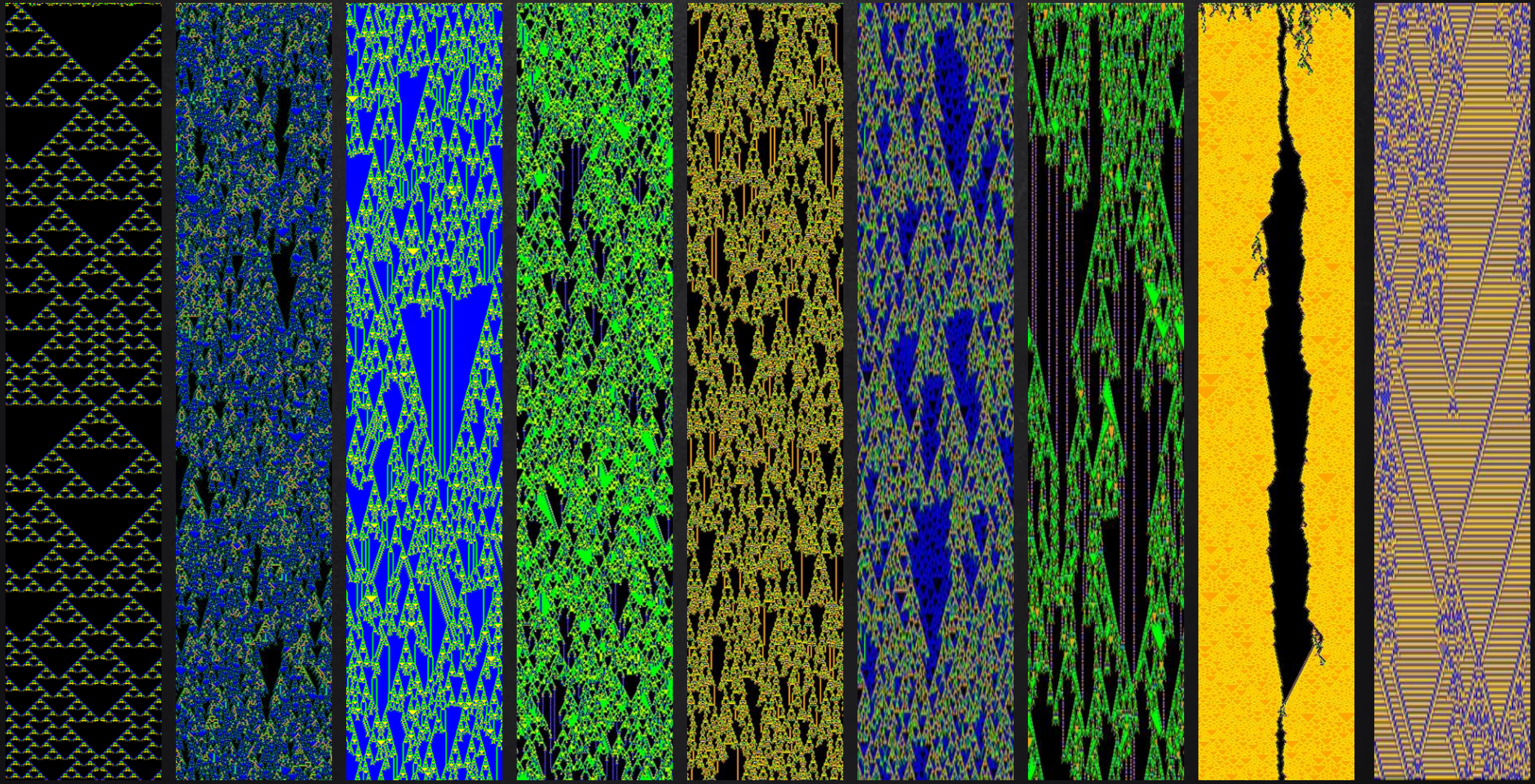
- A measure of the complexity or randomness of the automaton's state over time.

$$\sum_{k=1}^{N} \Pr\{s_k\}\, I\{sk\}$$

- Information:
  - Information increases as surprise increases.
  - Information decreases as probability increases.
  - Information is additive.
  - The information content of a message is proportional to the negative logarithm of its probability.

$$I\{s\} = -\lg \Pr\{s\}$$

$$H\{S\} = -\sum_{k=1}^{N} \Pr\{s_k\} \lg \Pr\{sk\}$$



Applications

Applications

Physics and Chemistry:

- Model physical and chemical processes, such as fluid dynamics, diffusion, reaction-diffusion systems, and crystal growth.

Biology:

- Simulate and analyze patterns in animal coats, the growth of plants and shells, the spread of diseases, neural networks, and the behavior of colonies of social insects like ants and bees.

Computer Science:

- Image processing, parallel computation, and machine learning. They can also be used to model internet traffic, simulate crowd behavior, and develop video game graphics and AI.

Cryptography

- Generate pseudorandom numbers and cryptographic keys, and for hashing functions.

Urban Planning and Social Science:

- Traffic flow, the growth of cities, and the spread and control of forest fires. They can also be used to model social dynamics and behaviors.

Ecology and Environmental Science:

- Model ecological interactions, population dynamics, and the spread of forest fires or diseases.

Open Issues

Open Issues

1. Scalability
   - Improve efficiency and computational cost of large-scale CA simulations

2. Boundary Conditions
   - Investigate impact of different boundary conditions in finite grids

3. Parameter Tuning
   - Determine optimal parameters for complex automata

4. Predictability
   - Study long-term behavior of chaotic and complex CA

5. Reversibility
   - Identify reversible CA and find their reverse rules

6. Model Validation
   - Ensure accurate representation of real-world systems

7. Universality
   - Investigate commonality of Turing completeness in CA

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Questions?

Questions (Revised)

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