

5a) minimize: $-5x_1 + 3x_2$ subject to:

$$\begin{aligned}x_1 - x_2 &\leq 1 \\ 2x_1 - x_2 &\leq 2 \\ x_1, x_2 &\geq 0\end{aligned}$$

convert to standard form -

maximize $5x_1 - 3x_2$ (same constraints)

slack form -

$$z = 5x_1 - 3x_2$$

$$x_3 = -x_1 + x_2 + 1$$

$$x_4 = -2x_1 + x_2 + 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

basic solution #1 - $(0, 0, 1, 2) \Rightarrow z = 0$

pivot 1: choose x_1, x_3

$$z = 5(1 + x_2 - x_3) - 3x_2 = 5 + 2x_2 - 5x_3$$

$$x_1 = 1 + x_2 - x_3$$

$$x_4 = -2(1 + x_2 - x_3) + x_2 + 2 = -x_2 + 2x_3$$

basic solution #2 - $(1, 0, 0, 0) \Rightarrow z = 5$

~~Q1~~ pivot 2: choose x_2, x_4

$$z = 5 + 2(2x_3 - x_4) - 5x_3 = 5 - x_3 - 2x_4$$

$$x_1 = 1 + (2x_3 - x_4) - x_3 = 1 + x_3 - x_4$$

$$x_2 = 2x_3 - x_4$$

Done!

basic solution #3 - $(1, 0, 0, 0) \Rightarrow z = 5$

So, $x_1 = 1, x_2 = 0$, minimum = -5 .

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