Parallelization of sorting algorithms

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Questions

1. Why were the first implementations of radix sort considered memory inefficient?
2. What is the difference between OpenMP and MPI?
3. Why is radix sort difficult to parallelize?

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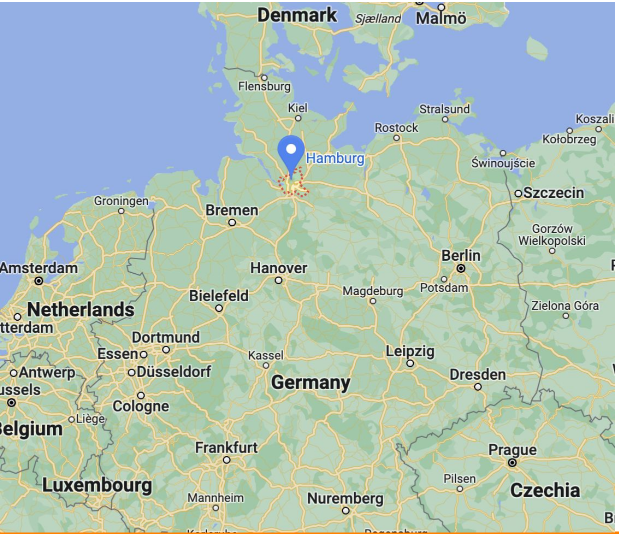

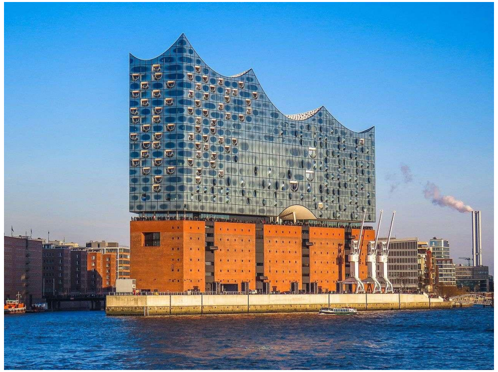

- Hiking, Traveling + Road trips, Cooking, Biking

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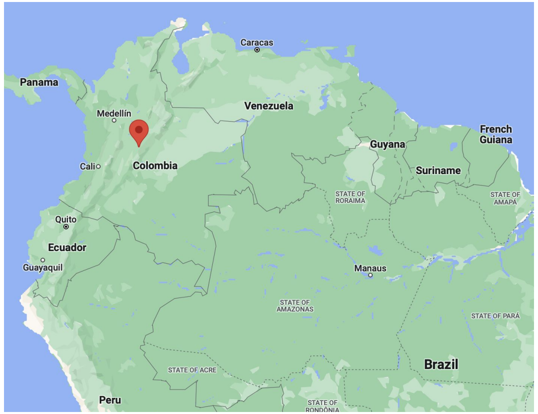


Bogotá: Capital of Colombia
Part of the Andes: 2,640 m (8,660 ft)
Average temp: 14.5 °C (58 °F)

- Food, Traveling, Music, Tenis, Climbing, Hiking, Diving

Outline

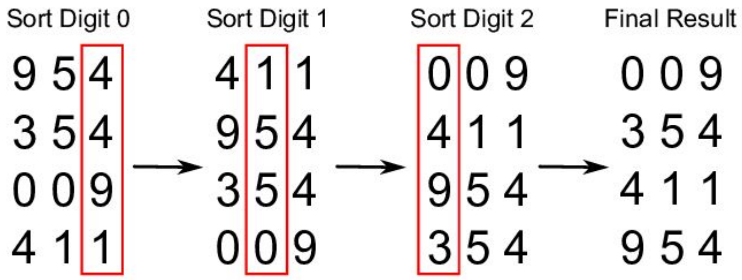
1. Overview:
   a. Brief review on Radix Sort
   b. Why is Radix Sort difficult to parallelize?
2. History
3. Algorithm:
   a. State of the art parallelization on GPU
   b. Parallelization on multi-core system (OpenMP and MPI)
4. Implementation:
   a. Our implementation of parallel Radix Sort
   b. Benchmarks
5. Applications
6. Open Issues

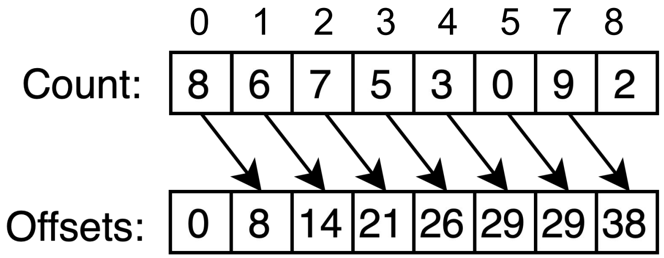
Overview: Radix sort

- Sorting takes place from LSD to HSD distributing elements into buckets
- Number of times sorting is performed depends on the radix
- Stable algorithm (required), not comparison-based sort, uses a version of counting sort
- Time complexity: O(d*n), d = # of digits, n = # of elements

Some definitions:

- Prefix sum: cumulative sum, used to compute bin-relative offsets
- GPU kernel: function that is executed in parallel (SIMD)





Tristram, Dale & Bradshaw, Karen. (2014). Identifying attributes of GPU programs for difficulty evaluation. South African Computer Journal. 53. 10.18489/sacj.v53i0.195.

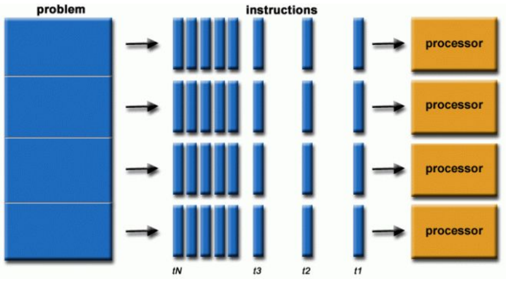
Overview: Radix sort

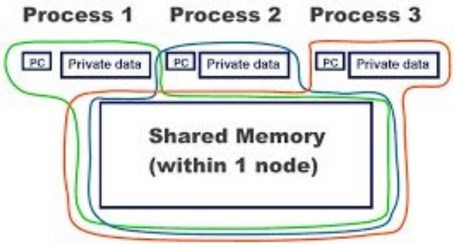
Why is Radix Sort hard to parallelize?

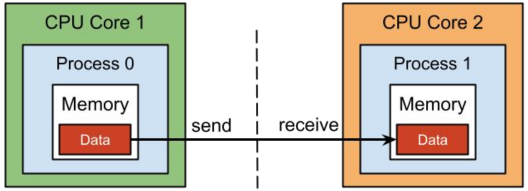
- Data dependencies: inherent to the algorithm
    - Thread cooperation
    - Ratio of computation to communication
- Irregular memory access patterns: moving records
- Parallelization involves overhead: dataset needs to be large enough to compensate

General challenges of parallelization:

- Amount of parallelizable work / Amdahl's Law
- Task granularity
- Load balancing
- Memory allocation and garbage collection (synchronization)
- Cache coherence
- Locality
- Control flow divergence







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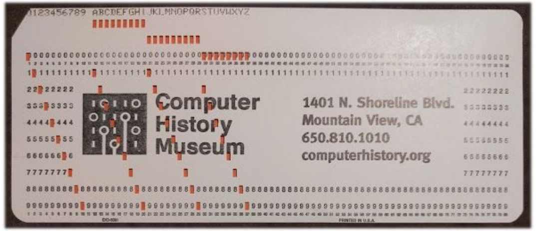
History

- 1887: **Herman Hollerith** (American Inventor)
  - Hollerith Machine for population count of 1890 U.S census.
  - 8 years to process (census every 10 years) vs 3 months.
- 1896: Hollerith founded the Tabulating Machine Company (TMC)
- 1900’s: Tabulating Machines went mainstream for accounting and tracking inventory
- 1911: After merger, TMC became Computing Tabulating Recording Company (CTR)
- 1924: CTR became International Business Machines Corporation (**IBM**).
- 1954: **Harold H. Seward** (MIT)
  - Creates first memory-efficient radix sort algorithm
  - Previously: allocated space for buckets of unknown size
  - Initial scan to get bucket sizes and offsets to do allocation
  - Invented counting sort and applied it to radix sort



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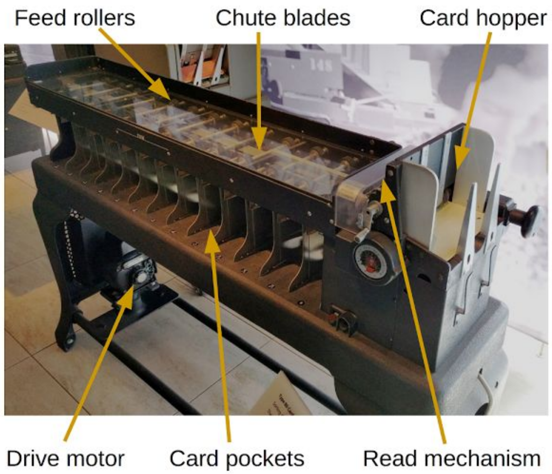
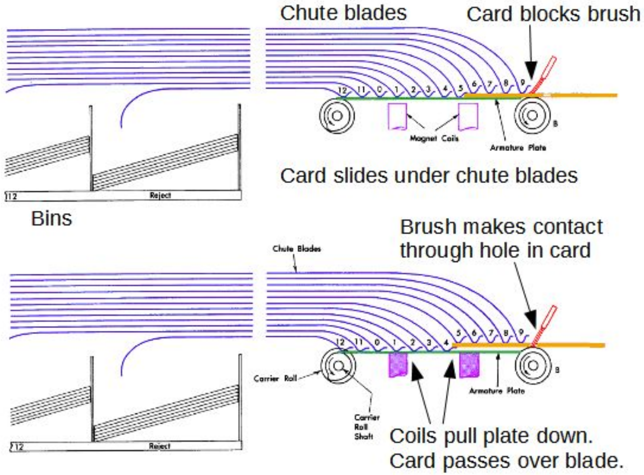
History

1. Wire brush (red) detects the presence or absence of a hole
2. If there is a hole in the card, the brush makes contact with the roller through the hole, completing the circuit
3. A stack of metal guides (chute blades) are used to direct the card into the appropriate bin





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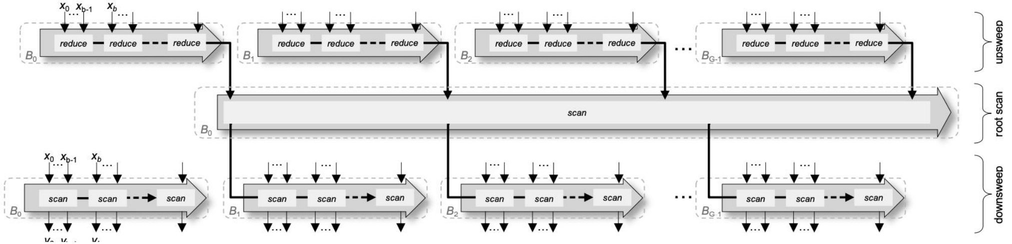
Onesweep: Radix Sort for GPUs

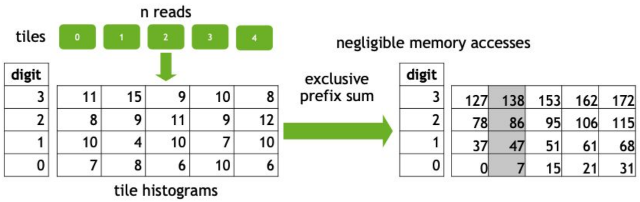
- Adinets and Merrill (2022) written in CUDA for NVIDIA GPUs.
- Uses single-pass prefix sum: reduces last-level memory traffic
  - **~2*n* vs 3*n* global memory operations**

Previous implementation: *reduce-then-scan,* 3 kernels and 2 full passes through the data

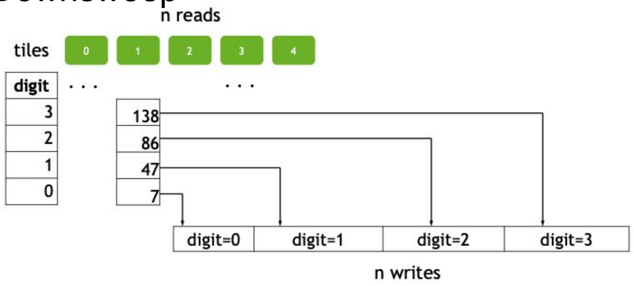
1. “upsweep” pass to compute per-block digit histograms
2. Prefix sum of block counts
3. “downsweep” pass to relocate keys



Upsweep



Downsweep



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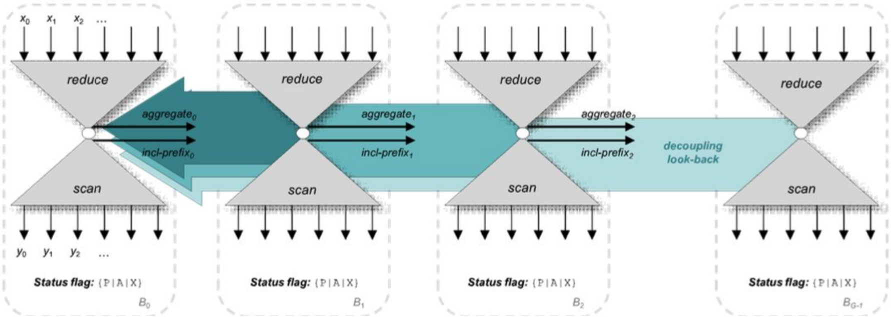
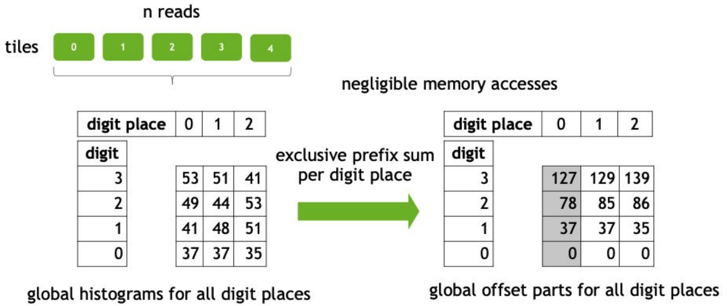
Onesweep: Radix Sort for GPUs

1. Histograms of global digit counts
2. Prefix sums of global digit counts
3. p = # of digits iterations of *chained scan* digit binning: each thread block reads its tile of elements, decodes key digits, participates in a chained scan of block-wide digit counts and scatters its elements into their global output bins (using its digit counts and global digit counts)

~(2*p*+1)*n vs* ~3*pn* memory operations for each digit-binning iteration.





digit place	0	1	2
digit			
3	53	51	41
2	49	44	53
1	41	48	51
0	37	37	35

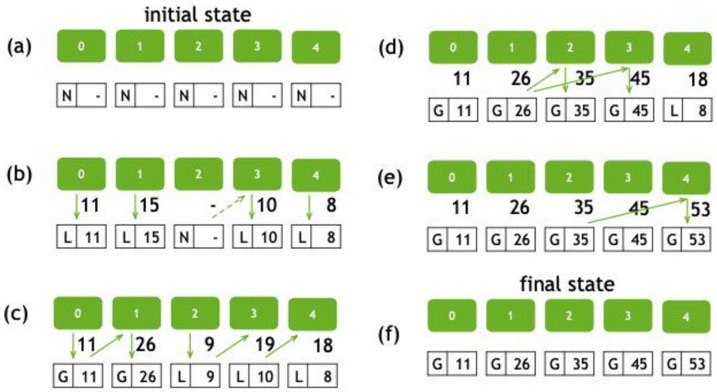
digit place	0	1	2
digit			
3	127	129	139
2	78	85	86
1	37	37	35
0	0	0	0

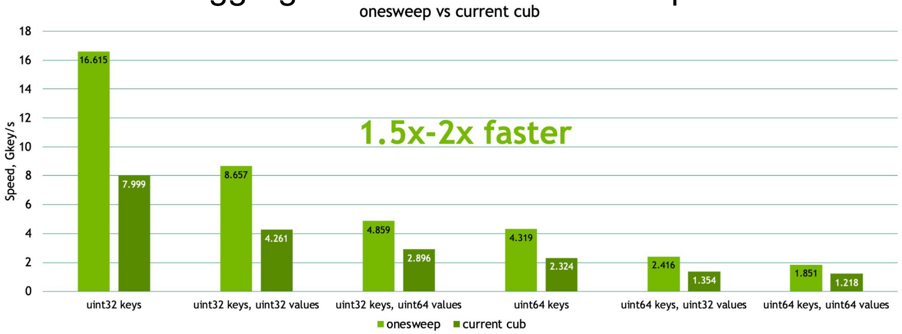
Adinets, Andy, and Duane Merrill. "Onesweep: A Faster Least Significant Digit Radix Sort for GPUs." arXiv, June 3, 2022. https://doi.org/10.48550/arXiv.2206.01784.

Onesweep: Radix Sort for GPUs

Chained scan:

- Each block is assigned a tile of the input and computes their local tile aggregate and the running prefix propagates from block to block.
- Latency in the propagation can be hidden via “decoupled lookback”: each block progressively consumes the per-tile totals of its predecessors until it discovers one that has also the global inclusive prefix.
- It must only wait on the processor to produce its local tile aggregate and all blocks are expected to





“We also reduce the number of binning iterations by using a larger digit size”

Adinets, Andy, and Duane Merrill. “Onesweep: A Faster Least Significant Digit Radix Sort for GPUs.” arXiv, June 3, 2022. https://doi.org/10.48550/arXiv.2206.01784.

Parallelization Techniques: MPI

- Message Passing Interface (MPI)
- Standard for passing messages across processes in a **distributed memory system**
- Standardisation started back in 1991, most recent: MPI 4
- Implemented by various vendors and open source efforts: OpenMPI, MPICH, IBM Spectrum, …
- High level: Run application multiple times concurrently and assign work to specific processes:

```
MPI_Comm_size(MPI_COMM_WORLD, &mpi_size);
MPI_Comm_rank(MPI_COMM_WORLD, &mpi_rank);
```

- Communicate data in case another process needs it:

Rank 0: `MPI_Send(buf, 1, MPI_INT, mpi_rank + 1, 0, MPI_COMM_WORLD);`
Rank 1: `MPI_Recv(buf, 1, MPI_INT, mpi_rank - 1, 0, MPI_COMM_WORLD);`

Parallelization Techniques: MPI - RDMA

- Remote direct memory access (RDMA) allows to access the memory of another process while bypassing both operating systems
- Instead of copying data from the network to some kernel space and then to the application memory, the data is copied directly to the memory region of the application
- Requires compatible networking hardware: Infiniband, advanced Ethernet
- No explicit synchronization: Application needs to be aware of where memory is modified
- Collectively allocate memory on each process:

```
MPI_Win_allocate(length * sizeof(int), sizeof(int), MPI_INFO_NULL,
MPI_COMM_WORLD, &shared_sorting, &win);
```

- Access other processes memory if needed:

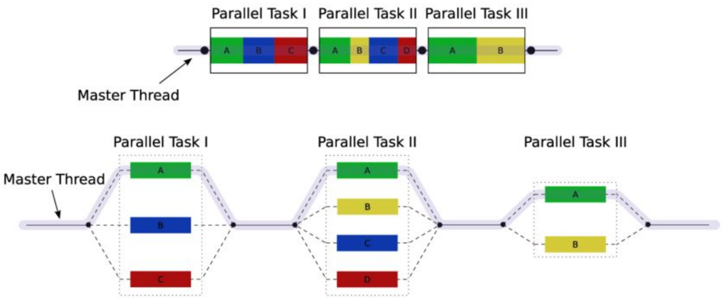
```
if (move_to_rank == mpi_rank)
    shared_sorting[rank_local_position] = output[i];
else {
    MPI_Put(&output[i], 1, MPI_INT, move_to_rank, rank_local_position, 1,
    MPI_INT, win); }
```

Parallelization Techniques: OpenMP

- Open Multi-Processing (OpenMP)
- API for **shared memory** parallelism and implemented within the compiler (GCC, Clang, …)
- Parallelism is achieved by the fork-join model where for specific parts of the application threads are created
- Implemented using compiler directives:

```
$ export OMP_NUM_THREADS=12

#pragma omp parallel for
for (int i = 0; i < length; i++) {
     output[i] = shared_sorting[i];
}
```



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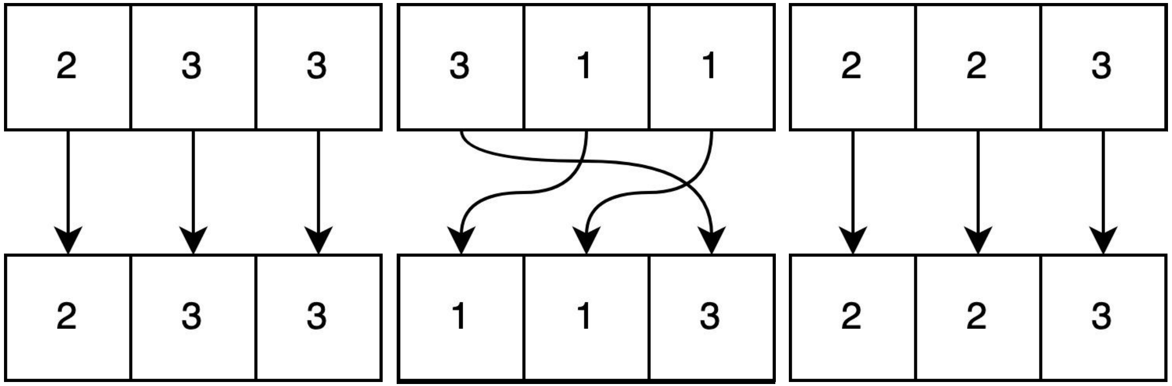
Parallelization Scheme

Bin

- # of Occurrences
- Cumulative count

Step 1: Process-local sorting

Process 1 | **Process 2** | Process 3



Counting

Bin	# of Occurrences
0	0
1	2
2	0
3	1

Cumulative

Bin	Cumulative count
0	0
1	0
2	2
3	2

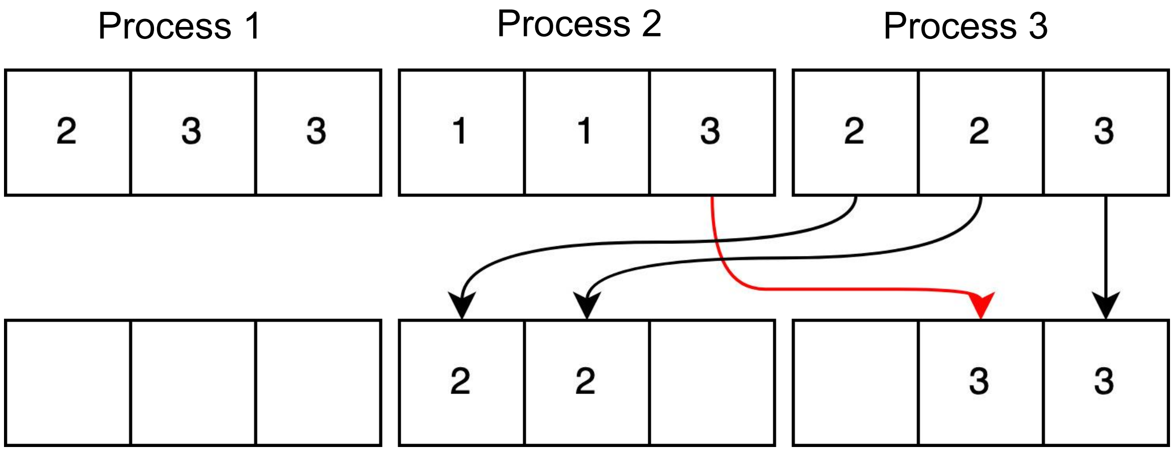
Step 2: Redistribute globally

Parallelization Scheme

Bin

- # of Occurrences
- Cumulative count

Step 2: Redistribute globally - **Non-parallelized**



Global Cumulative

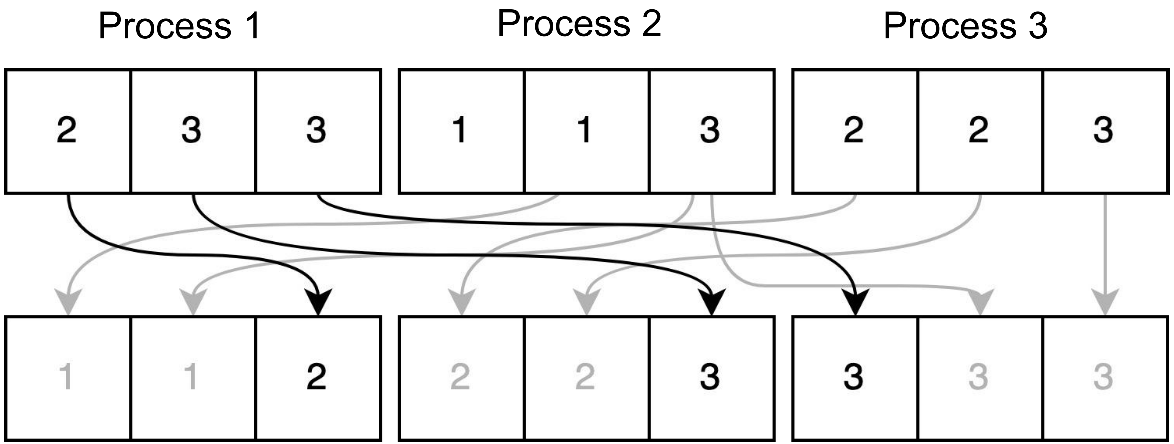
Bin	Cumulative count
0	0
1	2
2	5
3	9

Global Cumulative **Process 2**

Bin	Cumulative count
0	0
1	2
2	3
3	8

Parallelization Scheme

Step 2: Redistribute globally - **Parallelized**



Bin

- # of Occurrences
- Cumulative count

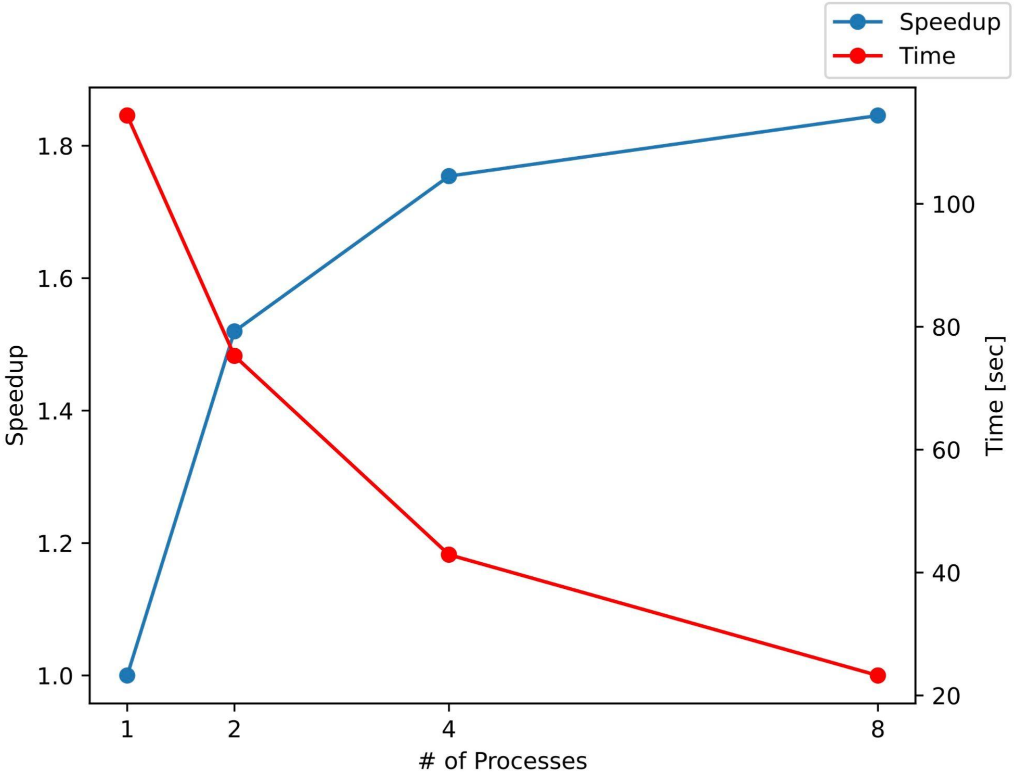
Offsetted Cumulative

Bin	Cumulative count
0	0
1	0
2	3
3	7

Strong Scaling

$$Speedup = t(1)/t(N)$$

- Strong scaling on 1, 2, 4 and 8 processes on Apple M2 Pro with OpenMPI@4.1.4
- Sorting 400.000.000 numbers
- Speedup trend noticeable to at least 8 processes



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Applications

- **In general:**
  - Search and query problems (to search DB efficiently)
  - Event-driven simulations
  - Schedulers
  - Huffman compression
  - Kruskal’s algorithm
  - Dijkstra's algorithm
  - Construction and manipulation of data structures, especially those modeling relationships in sparse systems
- **More common for radix sort:**
  - Sorting large datasets of integers
  - Text indexing (string processing)
  - Genome sequencing: “On the tandem duplication-random loss model of genome rearrangement”
    - Duplication loss step: equivalent to one step of radix sort.

Open Issues

- Challenge of achieving high levels of utilization from irregular workloads: which yield a very inconsistent performance response that is highly dependent upon the key distribution and problem size.
- Reducing communication overhead
- GPU parallelization: atomic read-modify-write operations can provide similar utility as prefix sum with less overhead, especially on architectures where they are implemented at the memory-level.
  But there are problems associated with it:
    - Their update-order is non-deterministic, preventing them from being used in stable LSD digit binning outside of simple digit-counting tasks
    - Performance can be unstable in sorting problems having low digit diversity due to contended accesses to the same counters
    - Their hardware support may be non-existent or have insufficient throughput for use at scale

References

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Discussion

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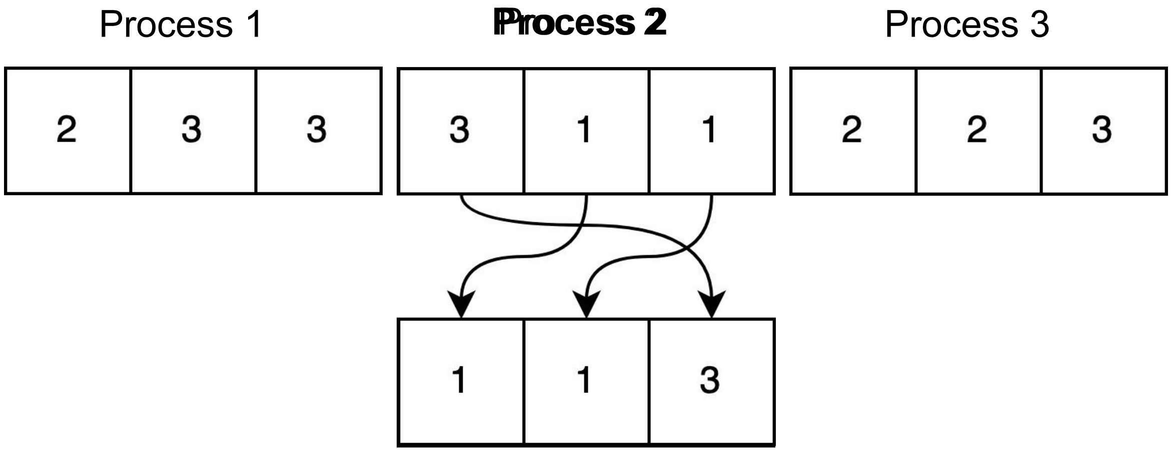
Parallelization Scheme



Bin

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- Cumulative count

Counting

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2	0
3	1

Cumulative

Bin	Cumulative count
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1	0
2	2
3	2