# N-body problems and simulation

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Alexander Krneta, Daniel Mishler

COSC581 - Algorithms

# Questions

1. Who first discovered the N-body problem?

2. Let's say we have a system of nonuniform density. Which method would have faster performance: Barnes-Hut or the Fast Multipole Method?

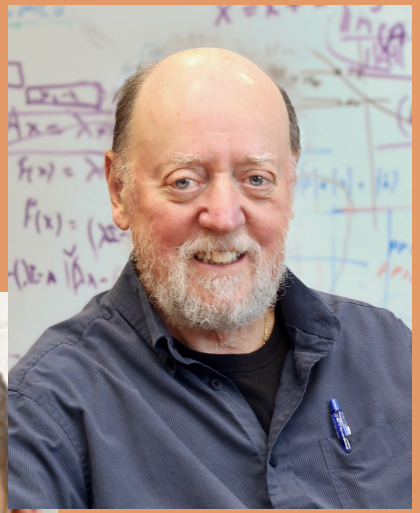
3. In a three-dimensional problem, neither Barnes-Hut nor the Fast Multipole Method would use a quadtree. What structure would they use instead?

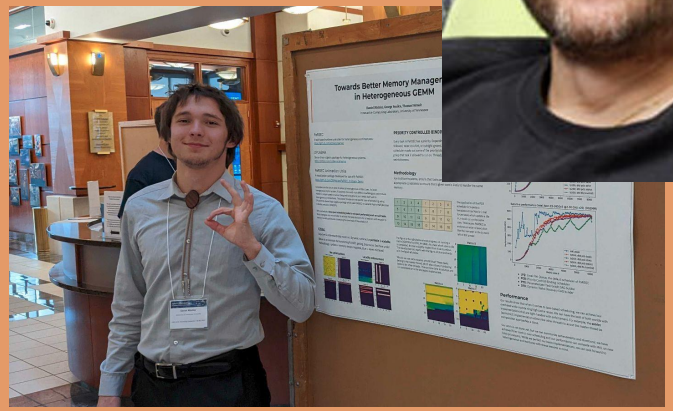
# Danny's city - Evansville

https://www.visitindiana.com
https://www.hyundaiofevansville.com

# ICL - DisCo

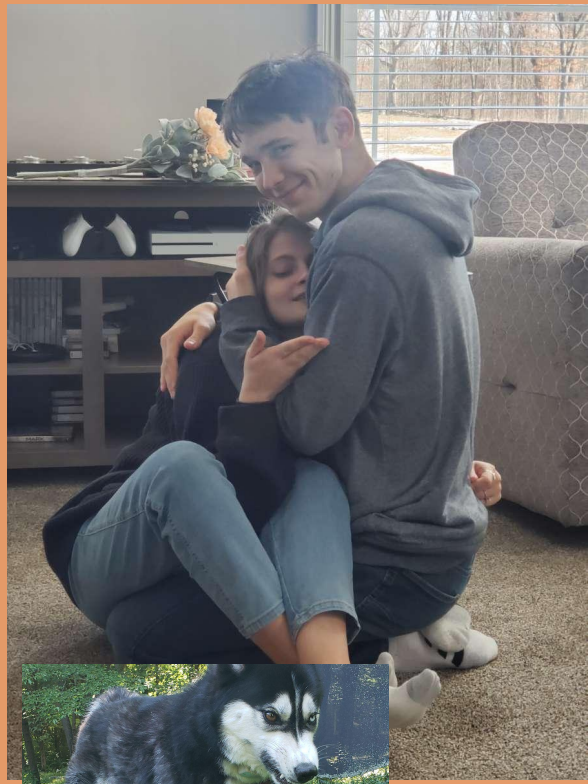
- Distributed Computing Group
- Task-based schedulers





# Danny Mishler

- Music
- Jiu Jitsu
- Just Married

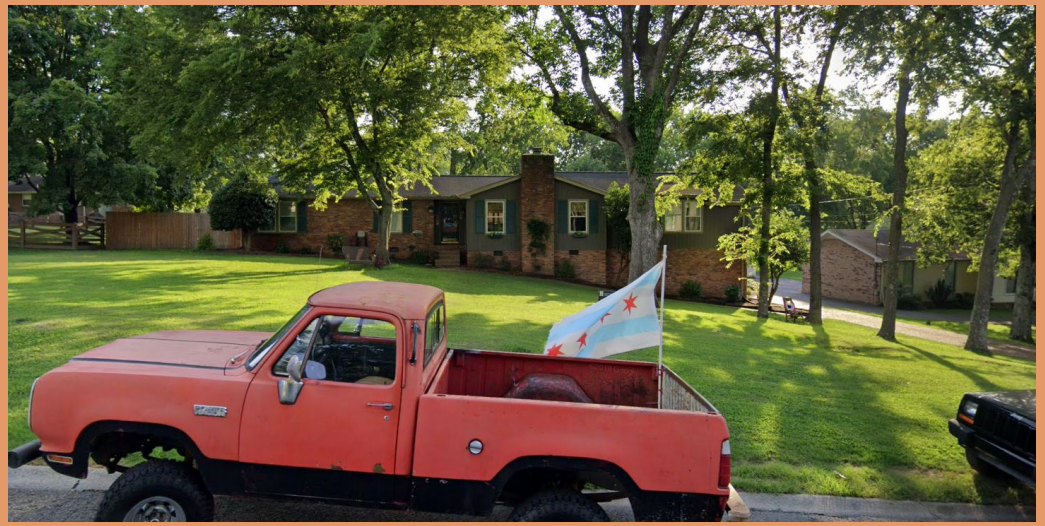



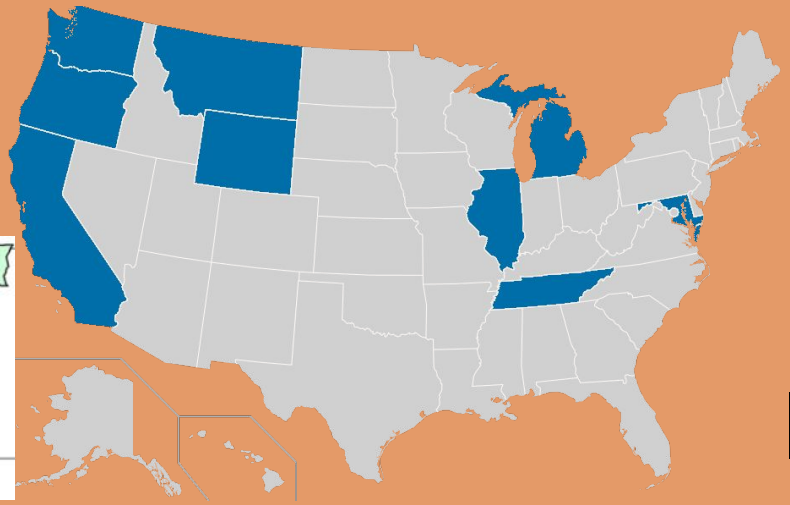
# Alexander Krneta

BS Physics - BS Computer Science

MS Computer Science

Moved 39 times (Military Brat)



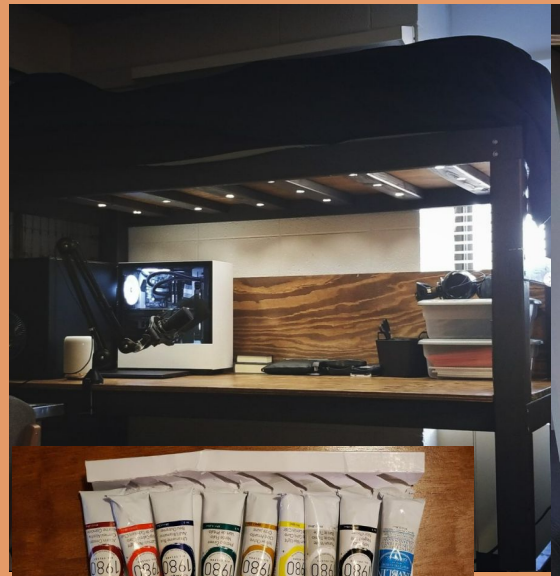
# Alexander Krneta

Christian

Maker

Reading

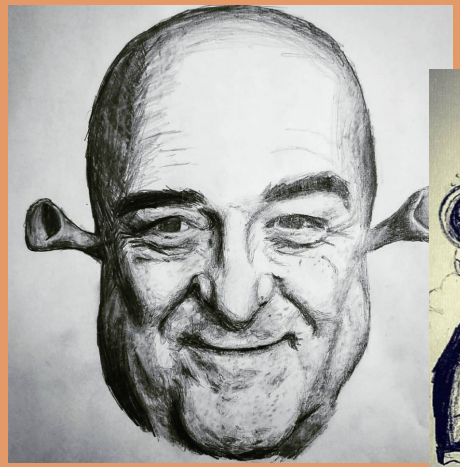
Music

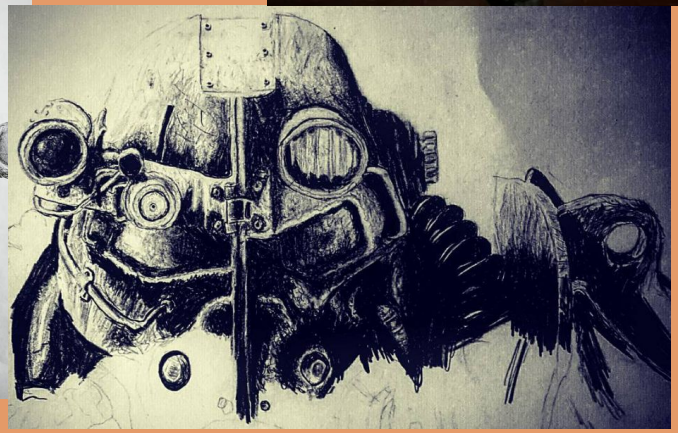


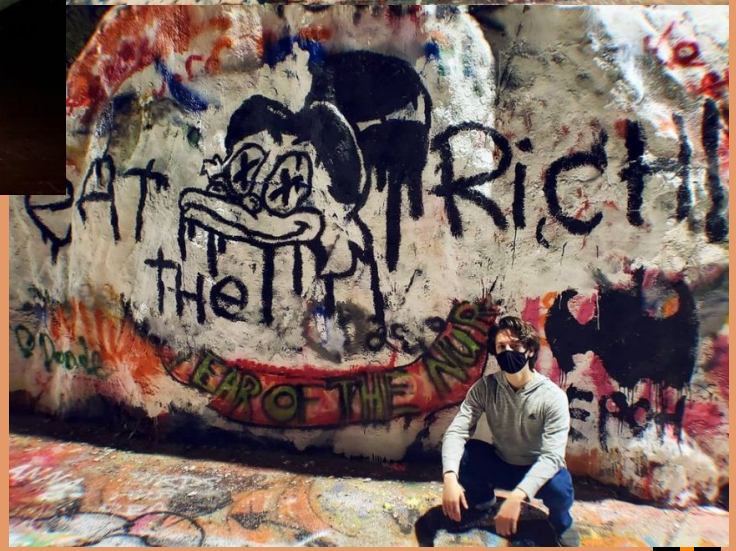
# Alexander Krenta

Art

Photography







# Alexander Krneta

Exploring

# Outline

- History
  - The N-body problem
- Overview
- Algorithms
  - FMM History and Implementation
  - Barnes-Hut History and Implementation
- Applications
- Implementations
- Open Issues

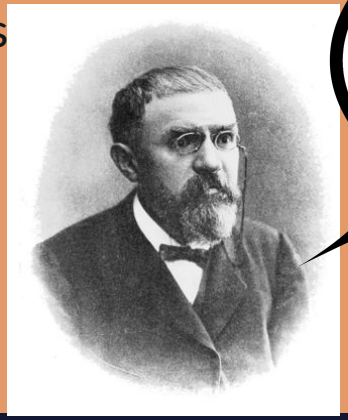
# 17th century: Newton predicts motion of planets

- Newton comes up with equations for planets orbits with three points (analytical geometry)
- But he's wrong!
  - Interplanet gravitational interactions affect orbits, too
- Newton concludes all bodies must attract each other, says it's unsolvable analytically, and moves on


Sir Isaac Newton: https://en.wikipedia.org/wiki/Isaac_Newton

# Late 19th century: Oscar II seeks a solution

- He was disappointed - no one came up with a solution
- Henri Poincaré won, but by doing something else
- The general solution was [illegible]he 1990s



Too hard - here's something else

https://en.wikipedia.org/wiki/Henri_Poincar%C3%A9

Money and fame for solving it!

Oscar II of Sweeden: https://en.wikipedia.org/wiki/Oscar_II

# Overview

- Problem: n objects are interacting with each other
  - Gravity
  - Charged particles
  - Cells
- N-body **problem**: Find the trajectories of *n* objects which obey Newtonian Physics given their initial configurations and the forces
- N-body **simulation**: Find a pseudo-solution to the N-body **problem** by numerically simulating the *n* objects and the forces between them

# N-body simulation: naive

```
Particle
{
    Mass;
    Position;
    Velocity;
    Force
}
```

```
One “timestep”
For each particle:
    Adjust velocity according to force
    Adjust position according to velocity
    Set force to 0

For each particle (a):
    For each particle (b):
        Adjust the force on (b) from (a)
```

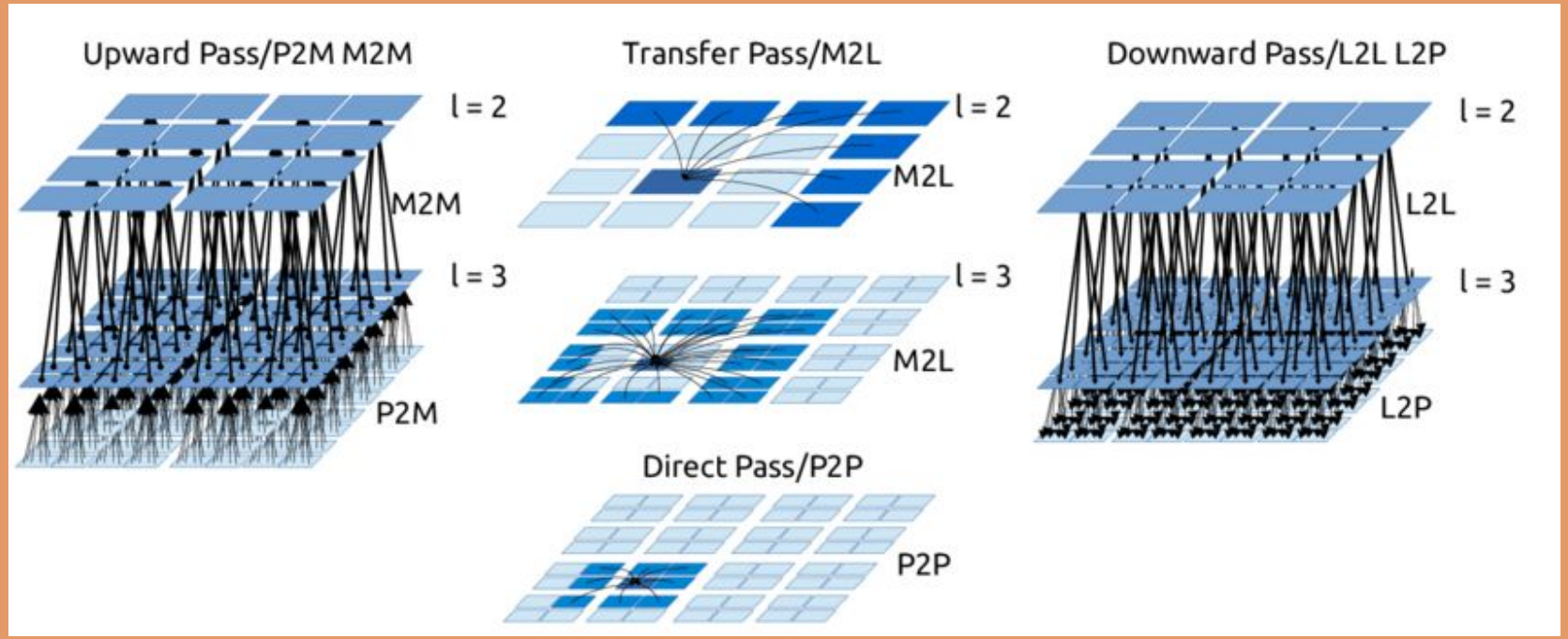
# History - FMM

- Expansion of Green's Function using Multipole Method
- Developed by Leslie F Greengard and Vladmir Rokhlin Jr.

- https://en.wikipedia.org/wiki/Leslie_Greengard
- https://seas.yale.edu/faculty-research/faculty-directory/vladimir-rokhlin

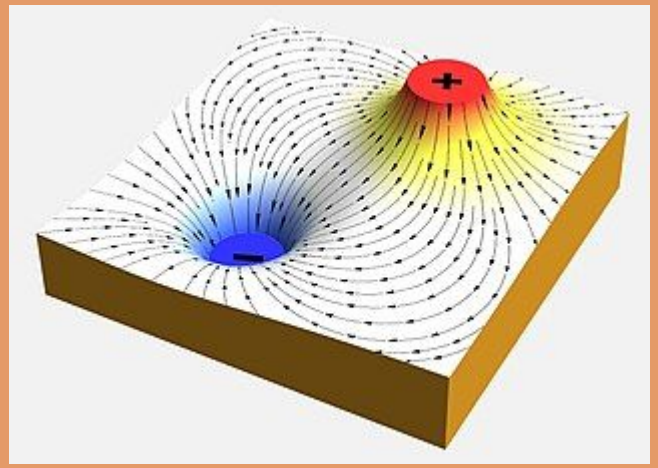
# FMM - an overview

- Use a Quadtree (balanced)

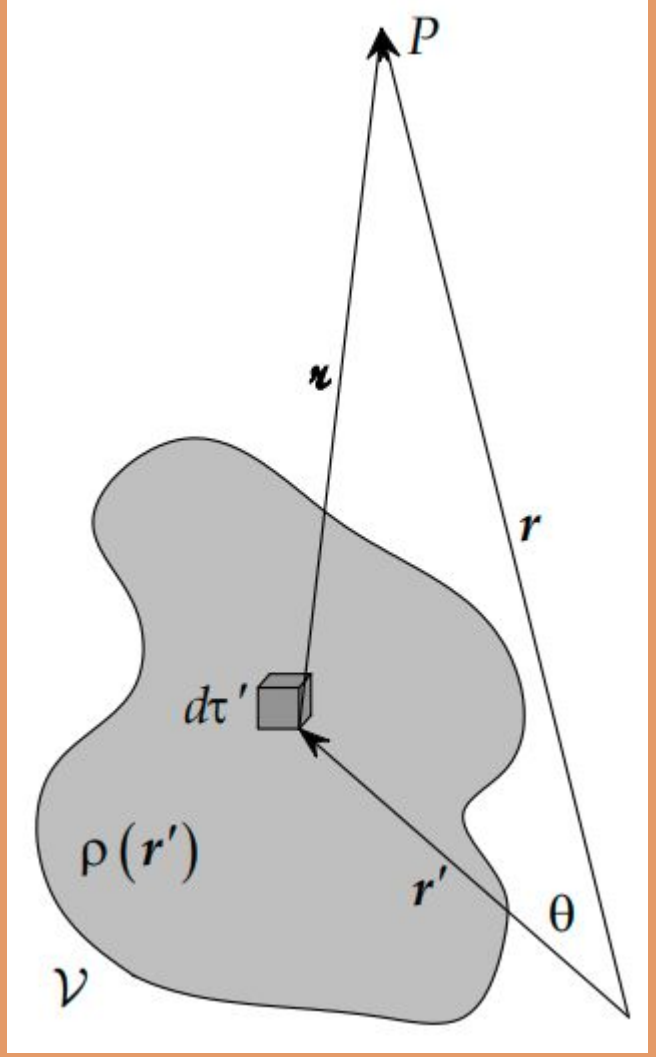


# What is a Multipole Expansion

- Mathematic Series (often of infinite terms)
  - Depending on angles, tells the force at a given point
  - Often, **very few** terms are needed for high accuracy
  - Relies on the Legendre Function



https://phys.libretexts.org/Bookshelves/Mathematical_Physics_and_Pedagogy/Mathematical_Methods/The_Multipole_Expansion



# Multipole Expansions

- Credit: David Harrison, University of Toronto

$$V_{\text{mon}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r}\int_{V'}\rho(\mathbf{r}')dV'$$

$$V_{\text{dip}}(\mathbf{r}) = -\frac{1}{4\pi\epsilon_0 r^2}\int_{V'}\rho(\mathbf{r}')\,(\hat{\mathbf{r}}\cdot\mathbf{r}')\,dV'$$

$$V_{\text{quad}}(\mathbf{r}) = \frac{1}{8\pi\epsilon_0 r^3}\int_{V'}\rho(\mathbf{r}')\left(3(\hat{\mathbf{r}}\cdot\mathbf{r}')^2 - r'^2\right)dV'$$

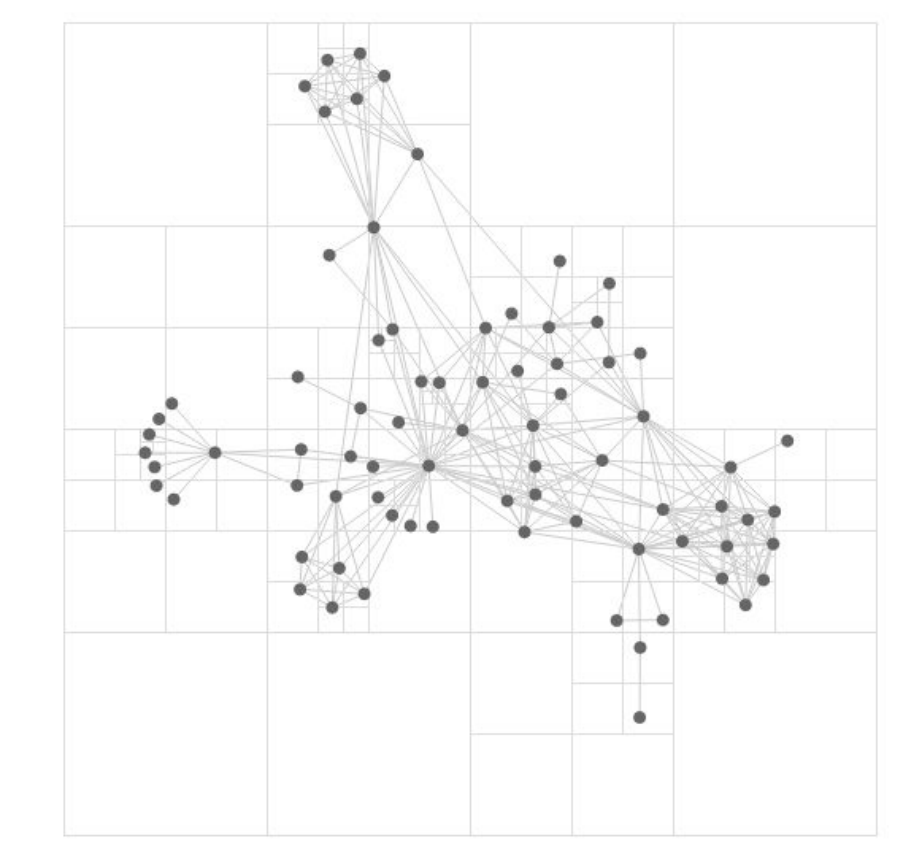
# History - Barnes-Hut

- Barnes-Hut
  - Josh Barnes and Piet Hut
  - https://en.wikipedia.org/wiki/Barnes%E2%80%93Hut_simulation
  - We don't know much about Barnes

Piet Hut, Dutch-American Astrophysicist

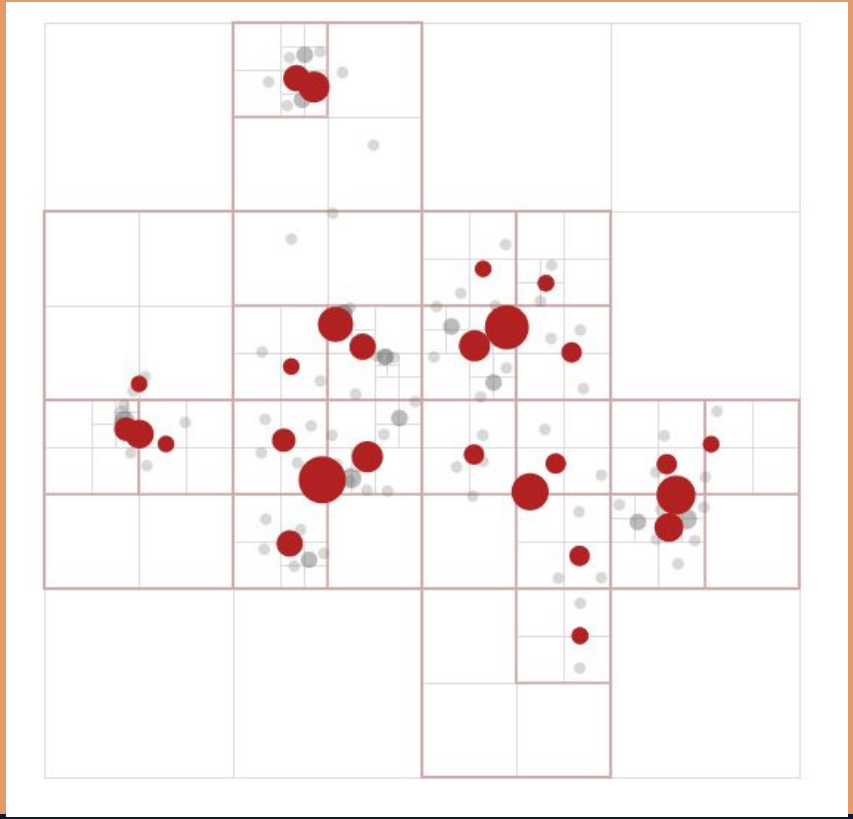
# Barnes-Hut: each point is in a box

- Also uses a quadtree, though not necessarily balanced



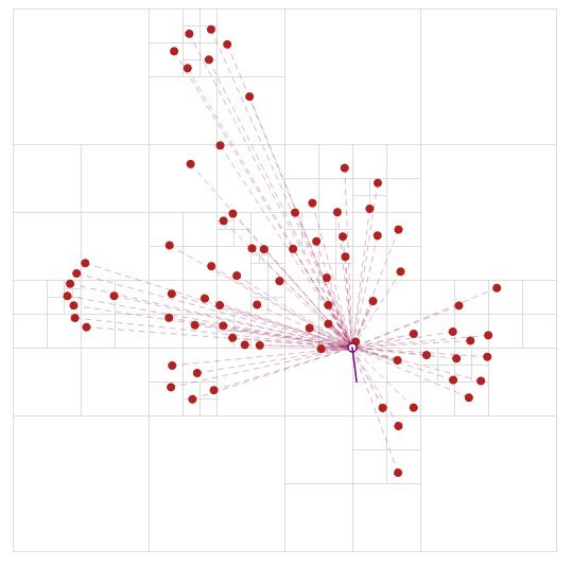
# Barnes-Hut: Each quadtree vertex has CM

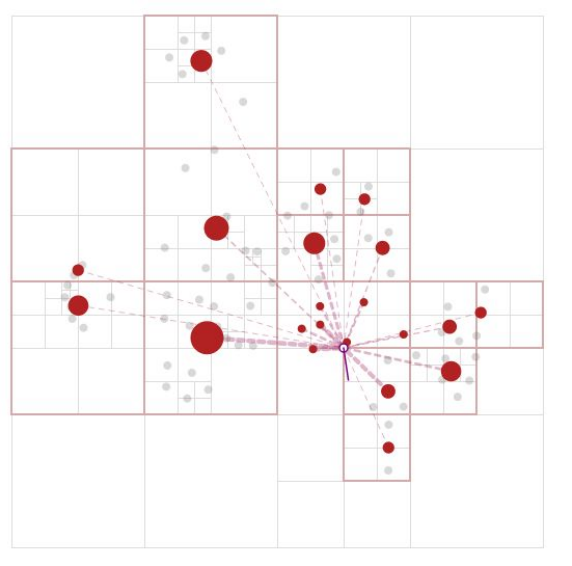
- Center of Mass is used to speed up calculations



# Barnes-Hut: Can adjust how much approximation

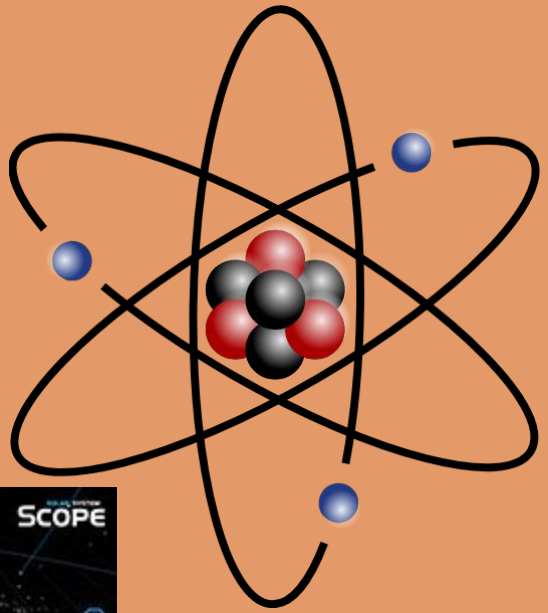
- Parameter Theta determines how far away a tree vertex must be until the approximation is used





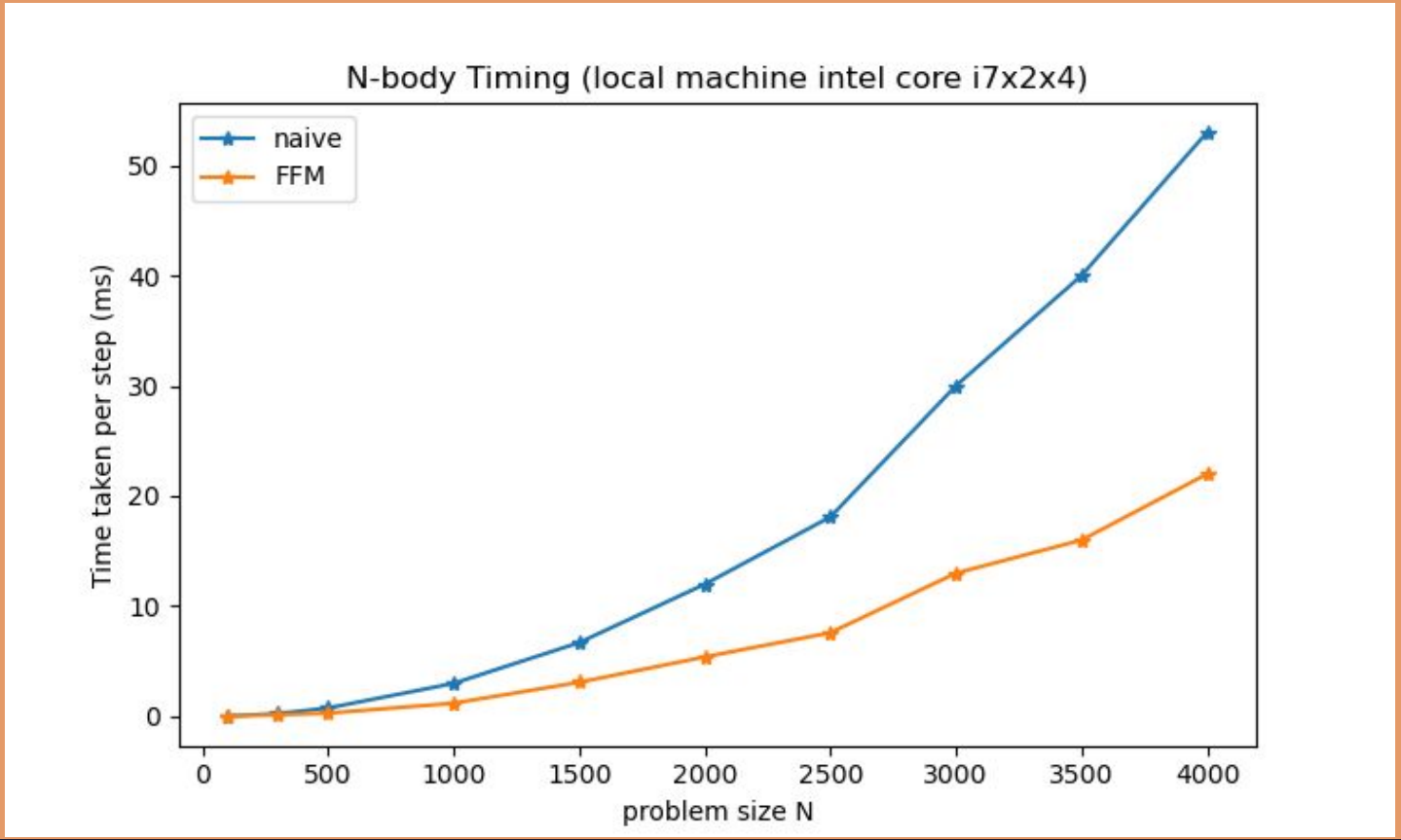
# Applications

- Planetary simulation
- Climate simulation
- Atomic simulation
- Fluid dynamics

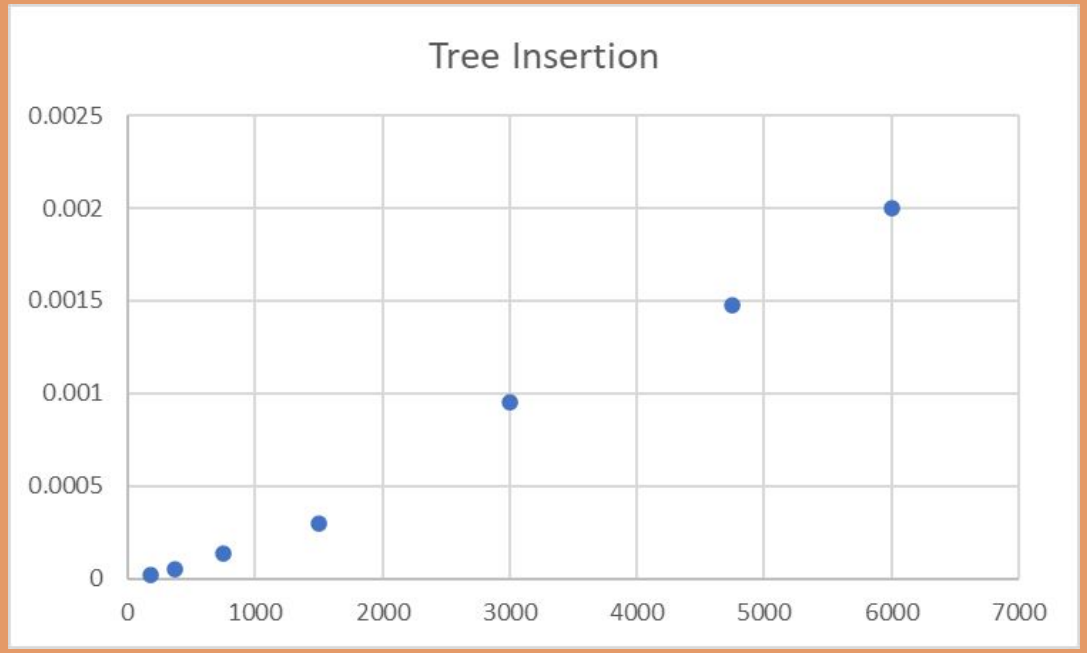



www.solarsystemscope.com

# Implementations - FMM



# Implementations - Barnes-Hut



# Comparisons of methods

- Naive: O(n^2)
- Barnes-hut: O(n ln(n))
  - best for nonuniform density)
- Fast Multipole Method: O(n ln(n))
  - best for uniform density

# Open Issues

- How can we simultaneously maximize fidelity and accuracy?
- What properties of a system can we exploit to improve our models?

# Questions

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# References


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- https://chat.openai.com/chat
- https://en.wikipedia.org/wiki/N-body_problem
- https://phys.libretexts.org/Bookshelves/Mathematical_Physics_and_Pedagogy/Mathematical_Methods/The_Multipole_Expansion