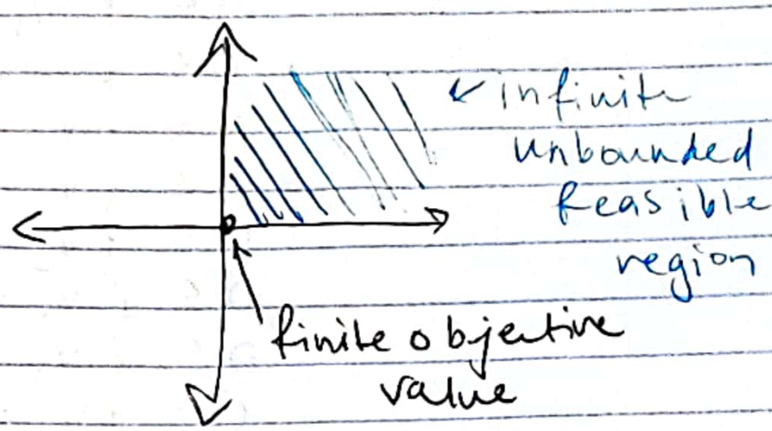
CS581 Homework #7 Solutions

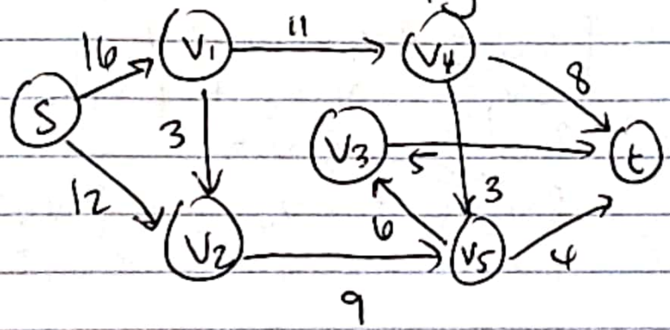
① Example:

minimize $x + y$, subject to:
$x, y \geq 0$

Hence,



② See textbook pg. 860-861



maximize: $\sum_{v \in V} f_{sv} - \sum_{v \in V} f_{vs}$,

subject to:

- $f_{uv} \leq c(u,v) \ \forall \ u,v \in V$
- $\sum_{v \in V} f_{vu} = \sum_{v \in V} f_{uv} \ \forall \ u \in V - \{s,t\}$
- $f_{uv} \geq 0 \ \forall \ u,v \in V$

Specifically,

maximize $f_{sv_1} + f_{sv_2}$,
subject to:

$$0 \le f_{sv_1} \le 16$$
$$0 \le f_{sv_2} \le 13$$
$$0 \le f_{v_1v_2} \le 3$$
$$0 \le f_{v_1v_4} \le 11$$
$$0 \le f_{v_2v_5} \le 9$$
$$0 \le f_{v_4v_5} \le 3$$
$$0 \le f_{v_5v_3} \le 6$$
$$0 \le f_{v_3t} \le 5$$
$$0 \le f_{v_4t} \le 8$$
$$0 \le f_{v_5t} \le 4$$
$$f_{sv_2} + f_{v_1v_2} = f_{v_2v_5}$$
$$f_{sv_1} = f_{v_1v_2} + f_{v_1v_4}$$
$$f_{v_5v_3} = f_{v_3t}$$
$$f_{v_2v_5} + f_{v_4v_5} = f_{v_5v_3} + f_{v_5t}$$
$$f_{v_1v_4} = f_{v_4v_5} + f_{v_4t}$$

③ a) minimize: $-5x_1 + 3x_2$, subject to:

$$x_1 - x_2 \leq 1$$
$$2x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

convert to standard form –

maximize $5x_1 - 3x_2$ (same constrainst)

Convert to slack form –

$$z = 5x_1 - 3x_2$$
$$x_3 = 1 - x_1 + x_2$$
$$x_4 = 2 - 2x_1 - x_2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

basic solution – $(x_1, x_2, x_3, x_4) = (0,0,1,2) \Rightarrow z=0$

pivot 1: $z = 5(1 + x_2 - x_3) - 3x_2 = 5 + 2x_2 - 5x_3$

$$x_1 = 1 + x_2 - x_3$$
$$\Rightarrow x_4 = 2 - 2(1 + x_2 - x_3) - x_2$$
$$= -3x_2 + 2x_3$$

basic solution – $(x_1, x_2, x_3, x_4) = (1,0,0,0) \Rightarrow z=5$

pivot 2: $z = 5 + 2\left(\frac{2}{3}x_3 - \frac{1}{3}x_4\right) - 5x_3 = 5 - \frac{11x_3}{3} - \frac{2x_4}{3}$

$$x_2 = \frac{2}{3}x_3 - \frac{1}{3}x_4$$

$$\Rightarrow x_1 = 1 + \left(\frac{2}{3}x_3 - \frac{1}{3}x_4\right) - x_3 = 1 - \frac{1}{3}x_3 - \frac{1}{3}x_4$$

basic solution – $(x_1, x_2, x_3, x_4) = (1,0,0,0) \Rightarrow z=5$

Done!

So, $x_1 = 1$, $x_2 = 0$, minimum $= -5$

b) maximize: $5x_1 + 4x_2 + 3x_3$, subject to:

$$2x_1 + 3x_2 + x_3 \le 5$$
$$4x_1 + x_2 + 2x_3 \le 11$$
$$3x_1 + 4x_2 + 2x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0$$

Convert to slack form:

$$z = 5x_1 + 4x_2 + 3x_3$$
$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$
$$x_5 = 11 - 4x_1 - x_2 - 2x_3$$
$$x_6 = 8 - 3x_1 - 4x_2 - 2x_3$$
$$x_1, x_2, x_3 \ge 0$$

basic solution - $(x_1, \dots, x_6) = (0,0,0,5,11,8) \Rightarrow z = 0$

pivot 1: $z = \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4$

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{11}{2}x_3 - \frac{11}{2}x_4$$
$$\Rightarrow x_5 = 1 + 5x_2 + 0x_3 + 2x_4$$
$$\Rightarrow x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

basic solution - $(x_1, \dots, x_6) = (\frac{5}{2}, 0, 0, 0, 1, \frac{1}{2}) \Rightarrow z = \frac{25}{2}$

pivot 2: $z = 13 - 3x_2 - x_4 - x_6$

$$x_3 = 1 + x_2 + 3x_3 - 2x_6$$
$$\Rightarrow x_5 = 1 + 5x_2 + 2x_4 + 0x_6$$
$$\Rightarrow x_1 = 2 - 2x_2 - 2x_4 + x_6$$

basic solution - $(x_1, \dots, x_6) = (2,0,1,0,1,0) \Rightarrow z = 13$

Done!

so, $x_1 = 2$, $x_2 = 0$, $x_3 = 1$, maximum $= 13$

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