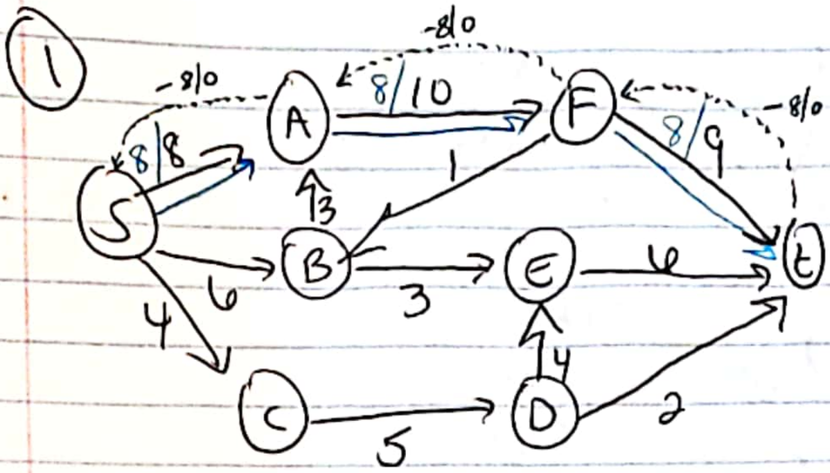
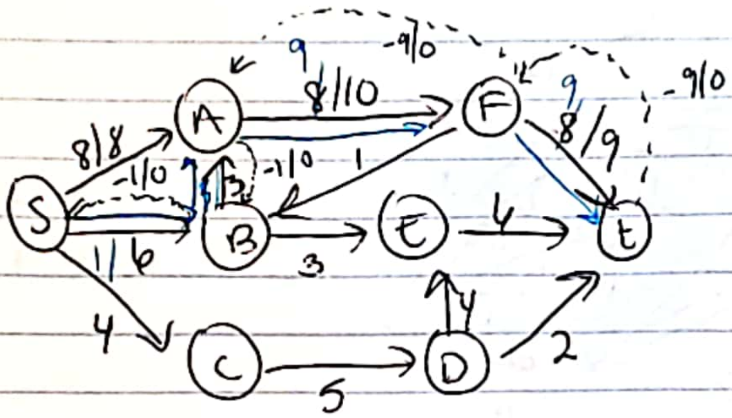
CS 581 Homework #6 Solutions

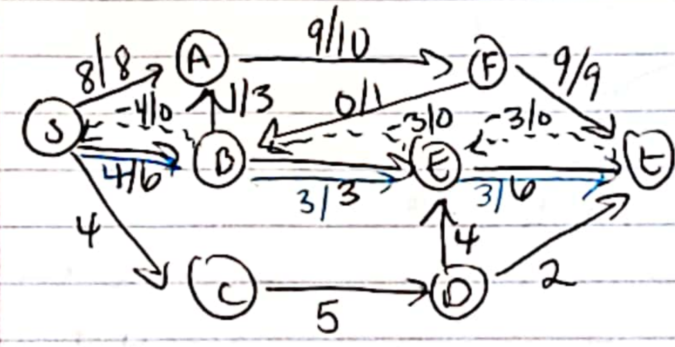
1



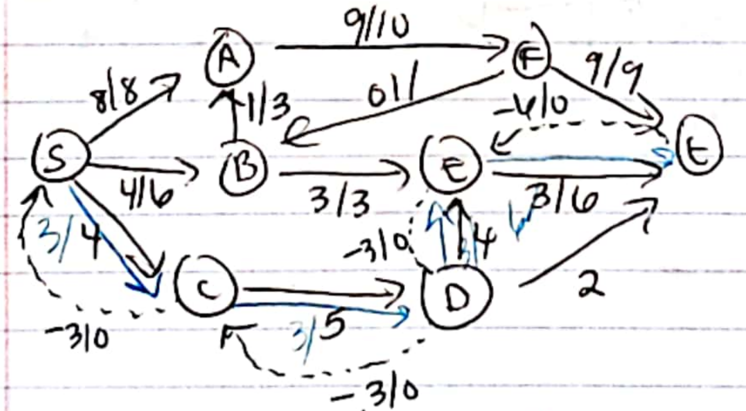
SAFt

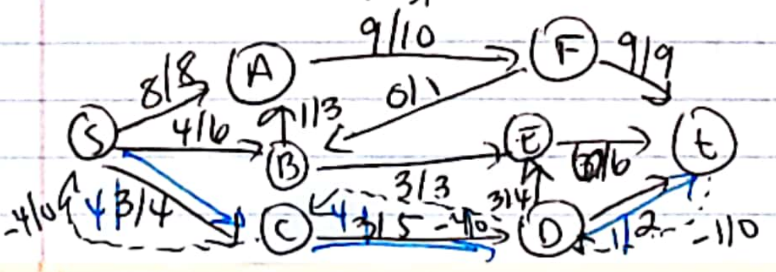


SAFt
SBAFt



SAFt
SBAFt
SBEt

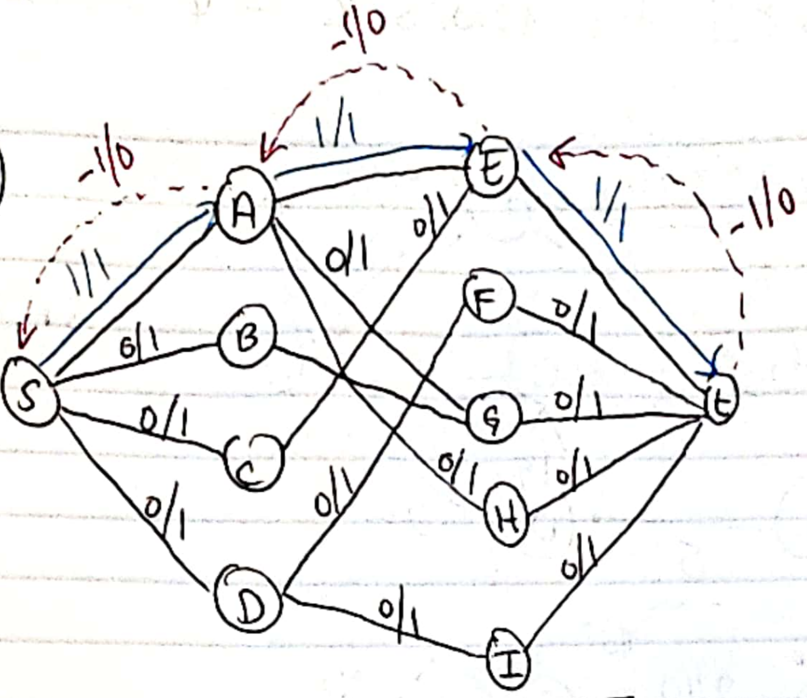




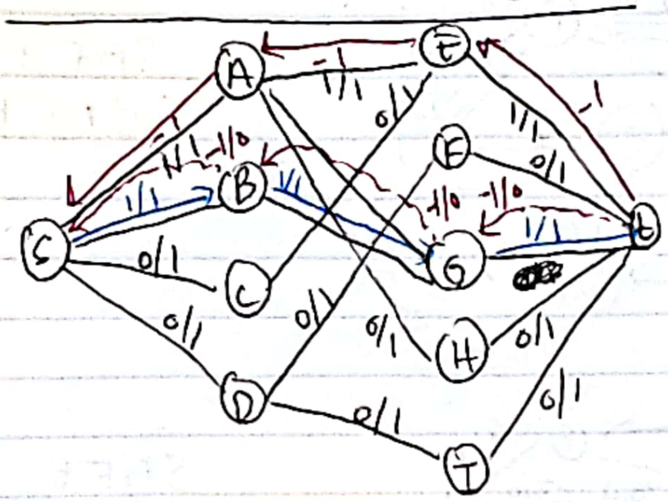
SAFt
SBAFt
SBEt
SCDEt
SCDt

max flow = 16

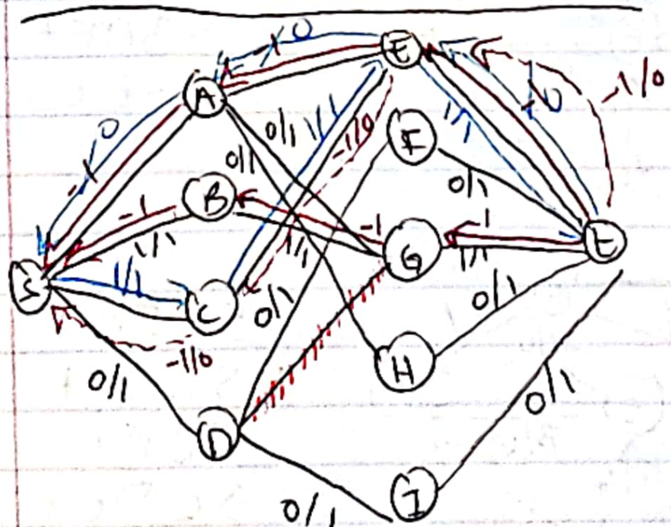
2



S A E t

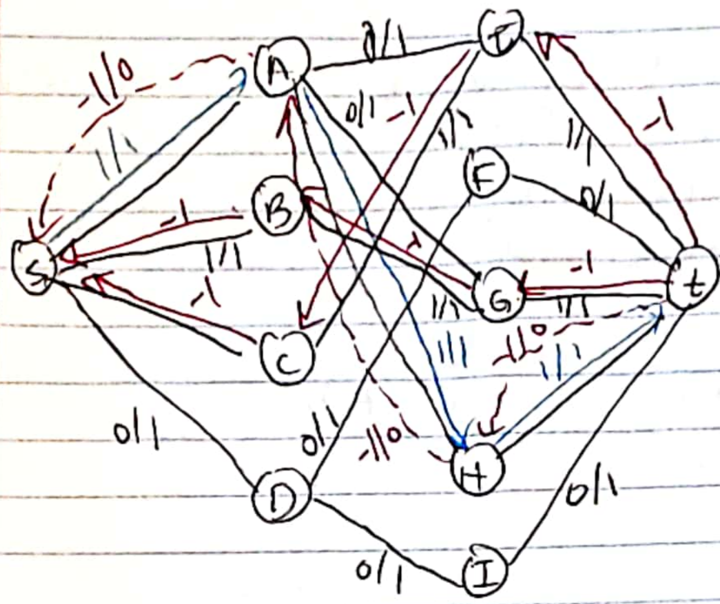


S A E t
S B G t

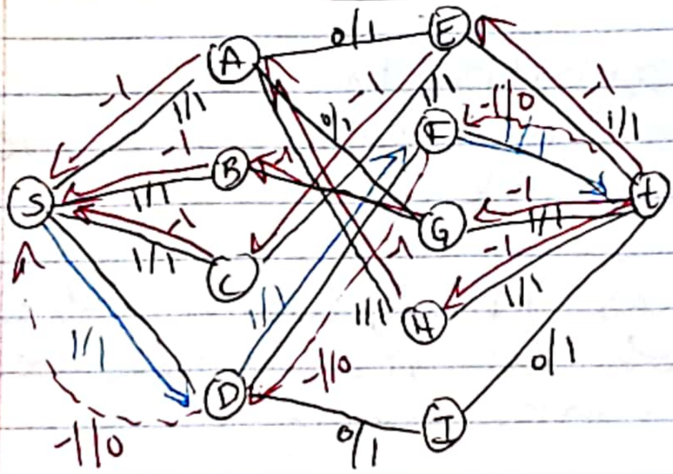


~~S A E t~~
S B G t
S C E t

(next page)



SBGt
SCEt
SAHt



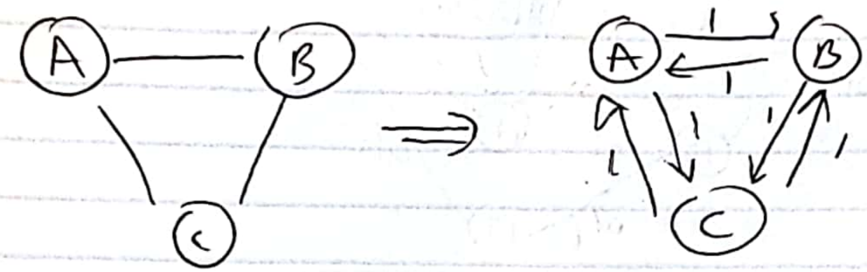
SBGt
SCEt
SAHt
SDFt

Algorithm terminates!

Final Bipartite matching:

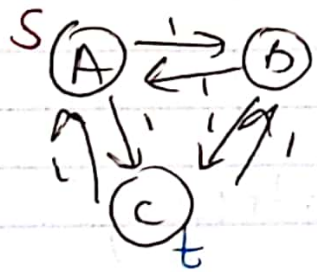
B → G
C → E
A → H
D → F (note I is alternate solution for F)

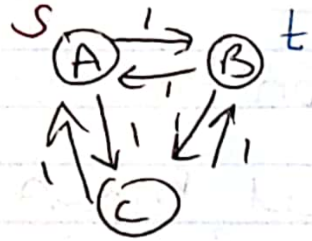
③ a) We first convert our undirected graph of $O(|E|)$ edges to a directed graph w/ 2 arcs w/ edge capacity 1.



We then arbitrarily fix the source, and allow the other $|V|-1$ vertices to be the sink.

Perform max flow, where the minimum of the max flows represents the edge connectivity.





b) proof:

Let $G = (V, E)$ be an arbitrary undirected graph.
Let $G' = (V, E')$ be G's bidirectional equivalent s.t.

$E' = \{c(u,v), c(v,u) = 1 \;\; \forall (u,v) \in E\}$

Let s be an arbitrary $v \in V$.
Let T be $(V - \{s\})$.

Let C be a set of k edges $\ni$ removing k disconnects G'.
Hence, $G' - C$ is the disconnected graph (S, T^*) where $s \in S$.

Then, for each $t \in T$ in G' find the max flow from s to t.

Hence, for at least one $t \in T$ in G', $t \notin S$, $t \in T^*$, and $t = k$. (by min cut)

$\square$