COSC581 - Algorithms
Spring 2023
Homework \#8

Due: Monday, 04/03/2023, at 5PM.

1. Let $\omega$ be an $n^{\text {th }}$ root of unity, and let k be a fixed integer. Evaluate:

$$
1+\omega^{\mathrm{k}}+\omega^{2 \mathrm{k}}+\cdots+\omega^{(\mathrm{n}-1) \mathrm{k}}
$$

2. Use the FFT to compute $C(\mathrm{x})$ as the product of $A(\mathrm{x})$ and $B(\mathrm{x})$, where $A(\mathrm{x})=\mathrm{x}^{2}+3 \mathrm{x}+1$ and $B(\mathrm{x})=\mathrm{x}+7$.
a. Find the value of $A(\mathrm{x})$ at the complex fourth roots of unity $(1,-1, \mathrm{i},-\mathrm{i})$.
b. Find the value of $B(\mathrm{x})$ at the complex fourth roots of unity.
c. Use the results of (a) and (b) to find the value of $C(\mathrm{x})$ at the complex fourth roots of unity.
d. Use these results to find the coefficients of $C(\mathrm{x})$.
3. What is the totient of 3044 ?
4. Consider an RSA crypto scheme with $\mathrm{n}=25$ and $\mathrm{D}=5$.
a. What is a possible value(s) of E ?
b. Encode two messages of your choosing.
c. Name three messages that are unencodable.
5. Given a finite simple undirected graph $G$ and a positive integer $k$, explain how you would reduce the problem of finding in $G$ an independent set of size $k$ to the problem of merely deciding whether such a set exists.
