COSC581 - Algorithms Spring 2023 Homework #8

Due: Monday, 04/03/2023, at 5PM.

- 1. Let  $\omega$  be an  $n^{th}$  root of unity, and let k be a fixed integer. Evaluate:  $1+\omega^k+\omega^{2k}+\cdots+\omega^{(n-1)k}$
- 2. Use the FFT to compute C(x) as the product of A(x) and B(x), where  $A(x) = x^2 + 3x + 1$ and B(x) = x + 7.
  - a. Find the value of A(x) at the complex fourth roots of unity (1, -1, i, -i).
  - b. Find the value of B(x) at the complex fourth roots of unity.
  - c. Use the results of (a) and (b) to find the value of C(x) at the complex fourth roots of unity.
  - d. Use these results to find the coefficients of  $C(\mathbf{x})$ .
- 3. What is the totient of 3044?
- 4. Consider an RSA crypto scheme with n=25 and D=5.
  - a. What is a possible value(s) of E?
  - b. Encode two messages of your choosing.
  - c. Name three messages that are unencodable.
- 5. Given a finite simple undirected graph G and a positive integer k, explain how you would reduce the problem of finding in G an independent set of size k to the problem of merely deciding whether such a set exists.