COSC581 - Algorithms
Spring 2023
Homework #6

Due: Tuesday, 03/21/2023, before class.

1. Consider the following asymmetric network adjacency matrix, in which a non-zero entry, x, in row i and column j represents an arc (directed edge) from vertex i to vertex j with weight x. Use the Ford-Fulkerson algorithm to maximize network flow. Specify the method you will use to find flow augmenting paths. At each iteration, you should draw the residual network, find such a path, and then draw the new network resulting from augmentation.

	S	A	B	C	D	E	F	t
S	0	8	6	4	0	0	0	0
A	0	0	0	0	0	0	10	0
B	0	3	0	0	0	3	0	0
C	0	0	0	0	5	0	0	0
D	0	0	0	0	0	4	0	2
E	0	0	0	0	0	0	0	6
F	0	0	1	0	0	0	0	9
t	0	0	0	0	0	0	0	0

2. Find a maximum bipartite matching given the adjacency list below. Formulate the problem as maximum network flow (where A-I are internal nodes in the graph and S, t represents the source and sink respectively). Show your work.

{A : [E,G,H], B : [G], C : [E], D : [F,I]}

3. The edge connectivity of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2.
    a. Show how to determine the edge connectivity of an undirected graph G = (V,E) by running a maximum-flow algorithm on at most |V| flow networks, each having O(V) vertices and O(E) edges.
    b. Prove that determining the edge connectivity with max-flow only requires choosing one arbitrary source node.