

CS581 HW #3 Solutions

① A) $T(n) = T(n/3) + 10$

$$\Rightarrow n^{\log_b a} = n^{\log_3 1} = n^0 = 1$$

Case 2:

$$f(n) = 10, \quad g(n) = n^{\log_b a} = 1$$

$$\lim_{n \rightarrow \infty} \frac{10}{1} = 10 \Rightarrow f = \Theta(1)$$

$$T(n) = \Theta(\lg n) \quad \square$$

B) $T(n) = 4T(n/4) + n \lg n$

$$\Rightarrow n^{\log_b a} = n^{\log_4 4} = n^1 = n$$

Case 2:

$$\lim_{n \rightarrow \infty} \frac{n \lg n}{n} = \lim_{n \rightarrow \infty} \lg n = \infty \Rightarrow \Omega \neq \Theta$$

not case 2!

Case 1: Let $\epsilon = 0.5 > 0$, then

$$\lim_{n \rightarrow \infty} \frac{n \lg n}{n^{1-0.5}} = \lim_{n \rightarrow \infty} n^{\frac{1}{2}} \lg n = \infty \Rightarrow \Omega \neq \Theta$$

not case 1!

So, between case 1 and 2
 \Rightarrow master theorem does not apply. \square

$$c) T(n) = 9T(n/3) + n^2 \log^3(n)$$

$$\Rightarrow n^{\log_b a} = n^{\log_3 9} = n^2$$

Case 2:

$$\lim_{n \rightarrow \infty} \frac{n^2 \log^3(n)}{n^2} = \lim_{n \rightarrow \infty} \log^3(n) = \infty \Rightarrow \Omega \neq \Theta$$

Case 3: Let $\epsilon = 1$, then

$$\lim_{n \rightarrow \infty} \frac{n^2 \log^3(n)}{n^{2+1}} = \lim_{n \rightarrow \infty} \frac{n^2 \log^3(n)}{n^3} = 0 \Rightarrow O \neq \Omega$$

not Case 2 or 3, so in between.
 \Rightarrow master theorem does not apply. \square

$$D) T(n) = 3T(n/6) + n^3$$

$$\Rightarrow n^{\log_b a} = n^{0.613...}$$

Case 3: Let $\epsilon = \log_6 12^{70}$, then

$$\lim_{n \rightarrow \infty} \frac{n^3}{n^{\log_6 3 + \log_6 12}} \frac{\lim_{n \rightarrow \infty} \frac{n^3}{n^2}}{n^2} = \lim_{n \rightarrow \infty} n = \infty \Rightarrow \Omega$$

regularity check: Let $c = \frac{1}{2} < 1$, then

$$3(n/6)^3 \leq \frac{1}{2} n^3 \quad \forall n \geq 1 \quad \checkmark$$

$$\text{So, } T(n) = \Theta(n^3)$$

\square

$$E) T(n) = T(n) + \log(n)$$

$a=1, b=1$, master theorem does not apply (requires $b > 1$).

$$F) T(n) = 2T(n/2) + \Theta(n^2)$$

$$\Rightarrow n^{\log_b a} = n^{\log_2 2} = n$$

Case 3: Let $\epsilon = 1/2$, then

$$\lim_{n \rightarrow \infty} \frac{\Theta(n^2)}{n^{\log_2 2 + \epsilon}} = \lim_{n \rightarrow \infty} \frac{\Theta(n^2)}{n^2} = 1 \Rightarrow \Omega$$

regularity check: Let $c = \frac{3}{4} < 1$, then

$$2(n/2)^2 \leq \frac{3}{4} n^2 \quad \forall n \geq 1 \quad \checkmark$$

So,

$$T(n) = \Theta(n^2)$$

□

② { 489, 783, 66, 1, 42, 88, 1092, 47, 68, 999 }

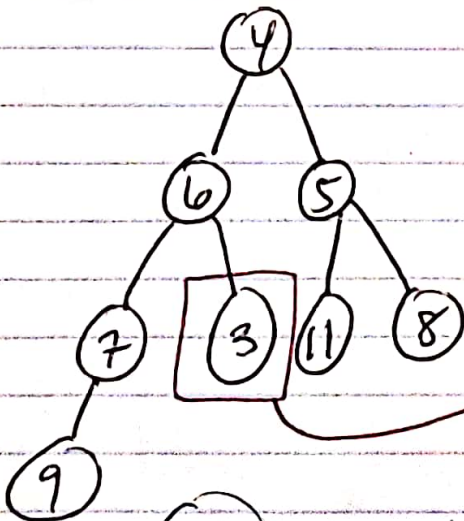
⇒ { 01, 42, 1092, 783, 66, 47, 88, 68, 489, 999 }

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⇒ { 1, 42, 47, 67, 68, 88, 489, 783, 999, 1092 }

③ { 4, 6, 5, 7, 3, 11, 8, 9 }



not currently a min heap b/c of #3

