CS581 HW#3 Solutions

① A) $T(n) = T(n/3) + 10$

$\Rightarrow n^{\log_b a} = n^{\log_3 1} = n^0 = 1$

<u>case 2</u>:

$f(n) = 10$, $g(n) = n^{\log_b a} = 1$

$\lim_{n\to\infty} \frac{10}{1} = 10 \Rightarrow f = \Theta(1)$

$T(n) = \Theta(\lg n)$

□

B) $T(n) = 4T(n/4) + n\lg n$

$\Rightarrow n^{\log_b a} = n^{\log_4 4} = n^1 = n$

<u>case 2</u>:

$\lim_{n\to\infty} \frac{n\lg n}{n} = \lim_{n\to\infty} \lg n = \infty \Rightarrow \Omega \neq \Theta$

not case 2!

<u>case 1</u>: Let $\varepsilon = 0.5 > 0$, then

$\lim_{n\to\infty} \frac{n\lg n}{n^{1-0.5}} = \lim_{n\to\infty} n^{\frac{1}{2}}\lg n = \infty \Rightarrow \Omega \neq O$

not case 1!

So, between case 1 and 2

$\Rightarrow$ master theorem does not apply.

□

C) $T(n) = 9T(n/3) + n^2 \log^3(n)$

$$\Rightarrow n^{\log_b a} = n^{\log_3 9} = n^2$$

Case 2:

$$\lim_{n\to\infty} \frac{n^2 \log^3(n)}{n^2} = \lim_{n\to\infty} \log^3(n) = \infty \Rightarrow \Omega \neq \Theta$$

Case 3: Let $\varepsilon = 1$, then

$$\lim_{n\to\infty} \frac{n^2 \log^3(n)}{n^{2+1}} = \lim_{n\to\infty} \frac{n^2 \log^3(n)}{n^3} = 0 \Rightarrow O \neq \Omega$$

not Case 2 or 3, so in between.
$\Rightarrow$ master theorem does not apply.

$\triangle$

D) $T(n) = 3T(n/6) + n^3$

$$\Rightarrow n^{\log_6 3} = n^{0.613\ldots}$$

Case 3: Let $\varepsilon = \log_6 12 > 0$, then

$$\lim_{n\to\infty} \frac{n^3}{n^{\log_6 3 + \log_6 12}} \lim_{n\to\infty} \frac{n^3}{n^2} = \lim_{n\to\infty} n = \infty \Rightarrow \Omega$$

regularity check: Let $c = \frac{1}{2} < 1$, then

$$3(n/6)^3 \leq \frac{1}{2} n^3 \quad \forall n \geq 1 \quad \checkmark$$

So, $T(n) = \Theta(n^3)$

$\square$

E) $T(n) = T(n) + \log(n)$

$a = 1$, $b = 1$, master theorem does not apply (requires $b > 1$).

F) $T(n) = 2T(n/2) + \Theta(n^2)$

$\Rightarrow n^{\log_b a} = n^{\log_2 2} = n$

Case 3: Let $\epsilon = 1 > 0$, then

$$\lim_{n\to\infty} \frac{\Theta(n^2)}{n^{\log_2 2+1}} = \lim_{n\to\infty} \frac{\Theta(n^2)}{n^2} = 1 \Rightarrow \Omega$$

regularity check: Let $c = \frac{3}{4} < 1$, then

$$2\left(\frac{n}{2}\right)^2 \leq \frac{3}{4} n^2 \quad \forall n > 1 \checkmark$$

So, $T(n) = \Theta(n^2)$

$\square$

② {489, 783, 66, 1, 42, 88, 1092, 47, 68, 999

⟹ {01, 42, 1092, 783, 66, 47, 88, 68, 489, 999}

⟹ {001, 042, 047, 067, 068, 783, 088, 489, 1092, 999}

⟹ {0001, 0042, 0047, 0067, 0068, 0088, 0489, 0783, 0999, 1092}

⟹ {1, 42, 47, 67, 68, 88, 489, 783, 999, 1092}

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③ {4, 6, 5, 7, 3, 11, 8, 9}