

CS581 HW#1 Solutions

- ① A) True, An algorithm must receive an input and produce an output by definition.
- B) False, Counterexample: an algorithm to produce the digits of π .
- C) True, see example for part B. By definition this algorithm is incorrect but still useful.
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② Proof:

base $n=1 \Rightarrow 5^1 + 15^1 = 20/10 \Rightarrow \text{True}$

step Suppose that $5^n + 15^n = 10j$ holds for some $n \geq 1$ and $j \in \mathbb{Z}$. Prove $n+1$:

$$5^{n+1} + 15^{n+1} = 5 \cdot 5^n + 15 \cdot 15^n$$

$$\text{(by dist)} = 5(5^n + 15^n) + 10(15^n)$$

$$\text{(by ind hyp)} = 5(10j) + 10 \cdot 15^n$$

$$= 10(5j + 15^n)$$

Since $5j + 15^n \in \mathbb{Z}$ then $10(5j + 15^n)$ is divisible by 10 by definition.

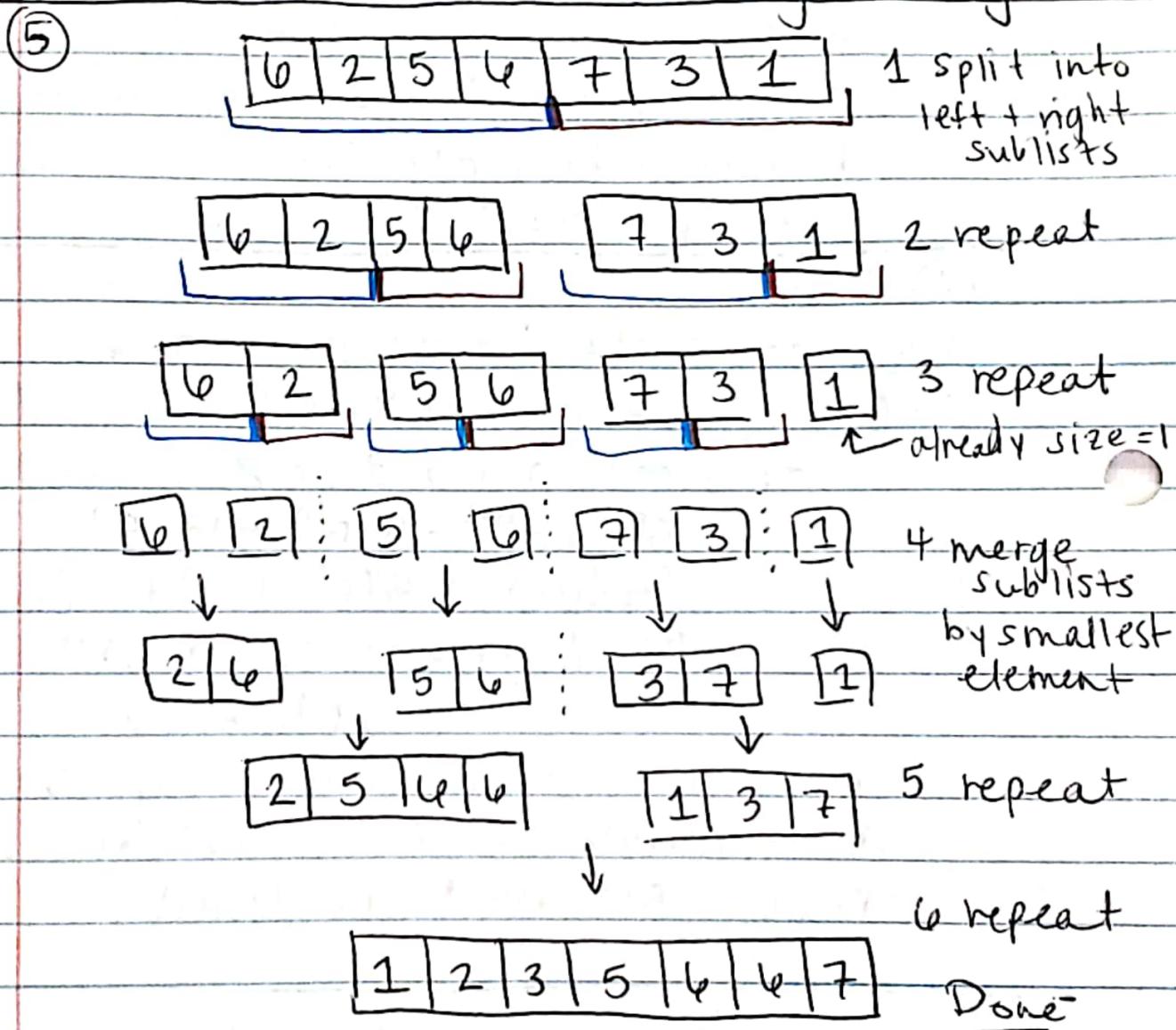
Thus, $5^{n+1} + 15^{n+1}$ is divisible by 10. \square

③ Let a, b, c be consecutive non-negative integers. Then $\exists k \in \mathbb{N} \rightarrow a = k-1, b = k, c = k+1$. Then,

$$a+b+c = (k-1) + k + (k+1) = 3k$$

Thus, $(a+b+c) / 3$ by definition. \square

④ In-place \Rightarrow does not require additional storage space to sort/swap elements.
Stable \Rightarrow Keeps duplicative elements in the same order during sorting.



⑥ A K_5 , contains a C_5 , and a C_5 contains a P_5 . Therefore, a P_5 will be most likely to occur.