

COSC581 - Algorithms  
Spring 2023  
Homework #1

Due: Tuesday, 01/31/2023, before class.

1. Answer **True** or **False** with justification:
  - a. An algorithm always has inputs and outputs.
  - b. An algorithm must terminate.
  - c. An incorrect algorithm is useful.
  
2. Use **induction** to prove that  $5^n + 15^n$  is divisible by 10 for all  $n \in \mathbb{N}$ .
  
3. Prove **without induction** that the sum of three consecutive, non-negative integers is always divisible by 3.
  
4. Define what it means for a sorting algorithm to be “in-place” and “stable” respectively.
  
5. Sort array  $\{6,2,5,6,7,3,1\}$  using **merge sort**. Show each step.
  
6. A little fun with pseudo-randomness\*. Suppose we start with an edgeless graph of order, say, 100. Then we begin uniformly generating edges (pairs of integers between 1 and 100) without replacement, stopping as soon as we produce in our graph a  $P_5$ , a  $C_5$ , or a  $K_5$  subgraph. Which of these three events is most likely and why?

*\*If this question seems too complex, Dr. Langston will be happy to explain it on Thursday.*