Supplementary Information for "Rapid evolution of reproductive barriers driven by sexual conflict" by S. Gavrilets.

PART 2.

Derivation of equations 9 from equations 7

Let $p(z)$ be the distribution of a quantitative trait z in the population with the mean $\overline{z}$ and central moments M_i $(= \int (z-\overline{z})^i p(z)dz)$. Let the fitness function be represented as a polynomial in $z - \overline{z}$

$$w(z) = a_0 + \sum_{i=1}^{k} a_i (z-\overline{z})^i,$$

where coefficients a_i are allowed to depend on the moments of $p(z)$ but not on z. Taking the mathematical expectation of both sides of the last equation one finds that the mean fitness $\overline{w}$ can be represented as

$$\overline{w} = a_0 + \sum_{i=2}^{k} a_i M_i.$$

Accordingly, the numerator in equation 7a is

$$\begin{aligned} I_1 &= \int z w(z) p(z) dz \\ &= \int (z - \overline{z} + \overline{z}) w(z) p(z) dz \\ &= \int (z-\overline{z})[a_0 + \sum_{i=1}^{k} a_i (z-\overline{z})^i] p(z) dz + \overline{z} \int w(z) p(z) dz \\ &= \sum_{i=1}^{k} a_i \int (z-\overline{z})^{i+1} p(z) dz + \overline{z}\, \overline{w} \\ &= \sum_{i=1}^{k} a_i M_{i+1} + \overline{z}\, \overline{w}. \end{aligned}$$

Plugging the above expressions for $\overline{w}$ and I_1 into equation (7a) one finds that the within generation change in $\overline{z}$ is

$$\begin{aligned} \Delta \overline{z} &= \overline{z}' - \overline{z} \\ &= \frac{I_1}{\overline{w}} - \overline{z} \end{aligned}$$

$$=\frac{\sum_{i=1}^{k} a_i M_{i+1}}{a_0 + \sum_{i=2}^{k} a_i M_i},$$

which is equation (9a).

In a similar way, the numerator in equation 7b is

$$\begin{aligned}
I_2 =& \int z^2 w(z) p(z) dz \\
=& \int (z - \overline{z} + \overline{z})^2 w(z) p(z) dz \\
=& \int \left[(z - \overline{z})^2 + 2(z - \overline{z})\overline{z} + \overline{z}^2 \right] w(z) p(z) dz \\
=& \int (z - \overline{z})^2 [a_0 + \sum_{i=1}^{k} a_i (z - \overline{z})^i] p(z) dz + 2\overline{z} \int (z - \overline{z}) w(z) p(z) dz + \overline{z}^2 \int w(z) p(z) dz \\
=& a_0 M_2 + \sum_{i=1}^{k} a_i M_{i+2} + 2\overline{z}(I_1 - \overline{z}\ \overline{w}) + \overline{z}^2\ \overline{w} \\
=& \sum_{i=0}^{k} a_i M_{i+2} + 2\overline{z}\ \sum_{i=1}^{k} a_i M_{i+1} + \overline{z}^2\ \overline{w}.
\end{aligned}$$

Thus,

$$\begin{aligned}
\Delta M_2 =& M_2' - M_2 \\
=& \frac{I_2}{\overline{w}} - (\overline{z}')^2 - M_2 \\
=& \frac{\sum_{i=0}^{k} a_i M_{i+2} + 2\overline{z}\ \sum_{i=1}^{k} a_i M_{i+1}}{\overline{w}} + \overline{z}^2 - (\overline{z} + \Delta\overline{z})^2 - M_2 \\
=& \frac{a_0 M_2 + \sum_{i=1}^{k} a_i M_{i+2} - M_2 \overline{w}}{\overline{w}} + 2\overline{z} \frac{\sum_{i=1}^{k} a_i M_{i+1}}{\overline{w}} - 2\overline{z}\Delta\overline{z} - (\Delta\overline{z})^2 \\
=& \frac{a_0 M_2 + \sum_{i=1}^{k} a_i M_{i+2} - M_2(a_0 + \sum_{i=1}^{k} a_i M_i)}{\overline{w}} - (\Delta\overline{z})^2 \\
=& \frac{\sum_{i=1}^{k} a_i (M_{i+2} - M_2 M_i)}{\overline{w}} - (\Delta\overline{z})^2,
\end{aligned}$$

which is equation (9b).

Analysis of equations 2 and 10

Taking the difference of equations (2a) and (2b) one finds that

$$\begin{aligned}\Delta(\overline{x}-\overline{y}) =& \alpha(\overline{x}-\overline{y})\left[2V_x\ \frac{s}{\theta}\ (1-\alpha\frac{(\overline{x}-\overline{y})^2}{\theta})-V_y\right]\\ =& \alpha(\overline{x}-\overline{y})2V_x\ \frac{s}{\theta}\ \left[1-\frac{1}{2}\frac{V_y}{V_x}\ \frac{\theta}{s}-\alpha\frac{(\overline{x}-\overline{y})^2}{\theta})\right].\end{aligned}$$

If condition 3 is satisfied, the expression in the square brackets is always negative and $u \equiv \overline{x}-\overline{y} \to 0$ asymptotically. If condition 3 is not satisfied, $u \to \pm\delta$ where δ is found from the quadratic equation obtained by equating the expression in the square brackets to zero.

At a mutation selection balance,

$$\Delta V_x = 0,\ \ \Delta V_y = 0.$$

From the second equation one finds the equilibrium value of V_y given by (6a). The equilibrium value of V_x is found using the fact that the expression in the square brackets above is zero:

$$\begin{aligned}0 =& 2\alpha V_x^2\ \frac{s}{\theta}\ \left(1-3\ \frac{\alpha(\overline{y}-\overline{x})^2}{\theta}\right)+\mu_x\\ =& 2\alpha V_x^2\ \frac{s}{\theta}\ \left(1-3(1-\frac{1}{2}\frac{V_y}{V_x}\ \frac{\theta}{s})\right)+\mu_x\\ =& -4\alpha\frac{s}{\theta}\ \left(V_x^2-\frac{3}{4}\frac{\theta}{s}V_y-\frac{1}{4}\frac{\theta}{s}\frac{\mu_x}{\alpha}\right)\end{aligned}$$

The last equation has the only positive solution given by (6b). From (6b) it follows that at mutation-selection balance equilibrium

$$\frac{1}{2}\ \frac{V_y}{V_x}\ \frac{\theta}{s} = \frac{4}{3\left(1+\sqrt{1+\frac{16m_x}{9\mu_y}\ \frac{s}{\theta}}\right)},$$

which is always $< 2/3$. Thus, condition (3) is never satisfied.